WP 4: Computation

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spectroscopy

expectation-maximization

model equation: $$y = H \cdot x + b$$

DRM and bremsstrahlung: $$H = \frac{\bar{n}V}{4\pi R^2} D \cdot Q.$$

likelihood: $$p(y|x) = \prod_{i \in R} \frac{e^{-(Hx+b)_i}}{y_i!} (Hx + b)_i^{y_i}$$

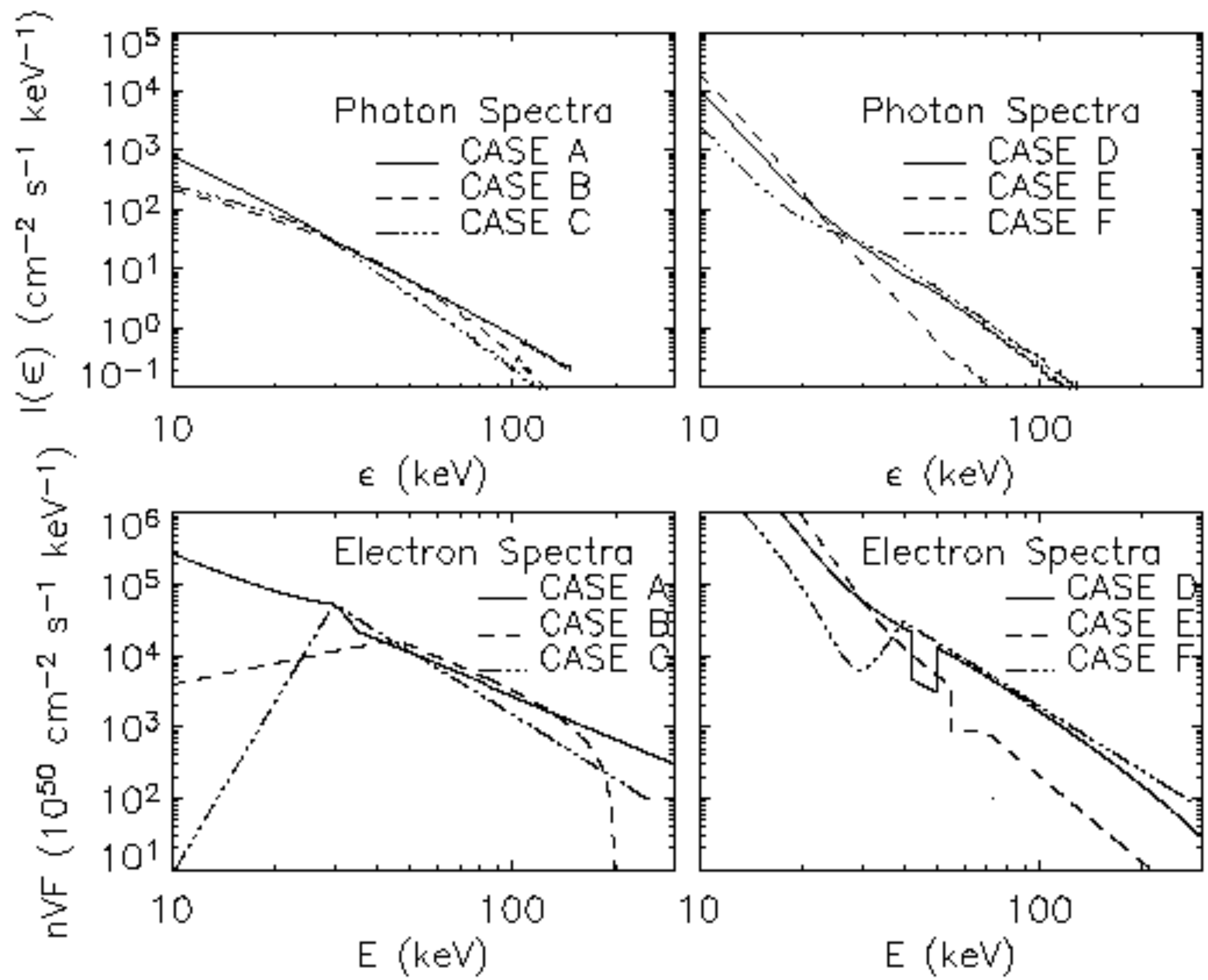
discrepancy: $$KL(y, x) = \sum_{i \in R} y_i \log \frac{y_i}{(Hx + b)_i} + (Hx + b)_i - y_i$$

constrained optimization: $$\min KL(y, x) \quad | \quad x \geq 0$$

successive approximation: $$x^{k+1} = \frac{x^k}{H^T \mathbf{I}} H^T \left(\frac{y}{Hx^k + b} \right)$$

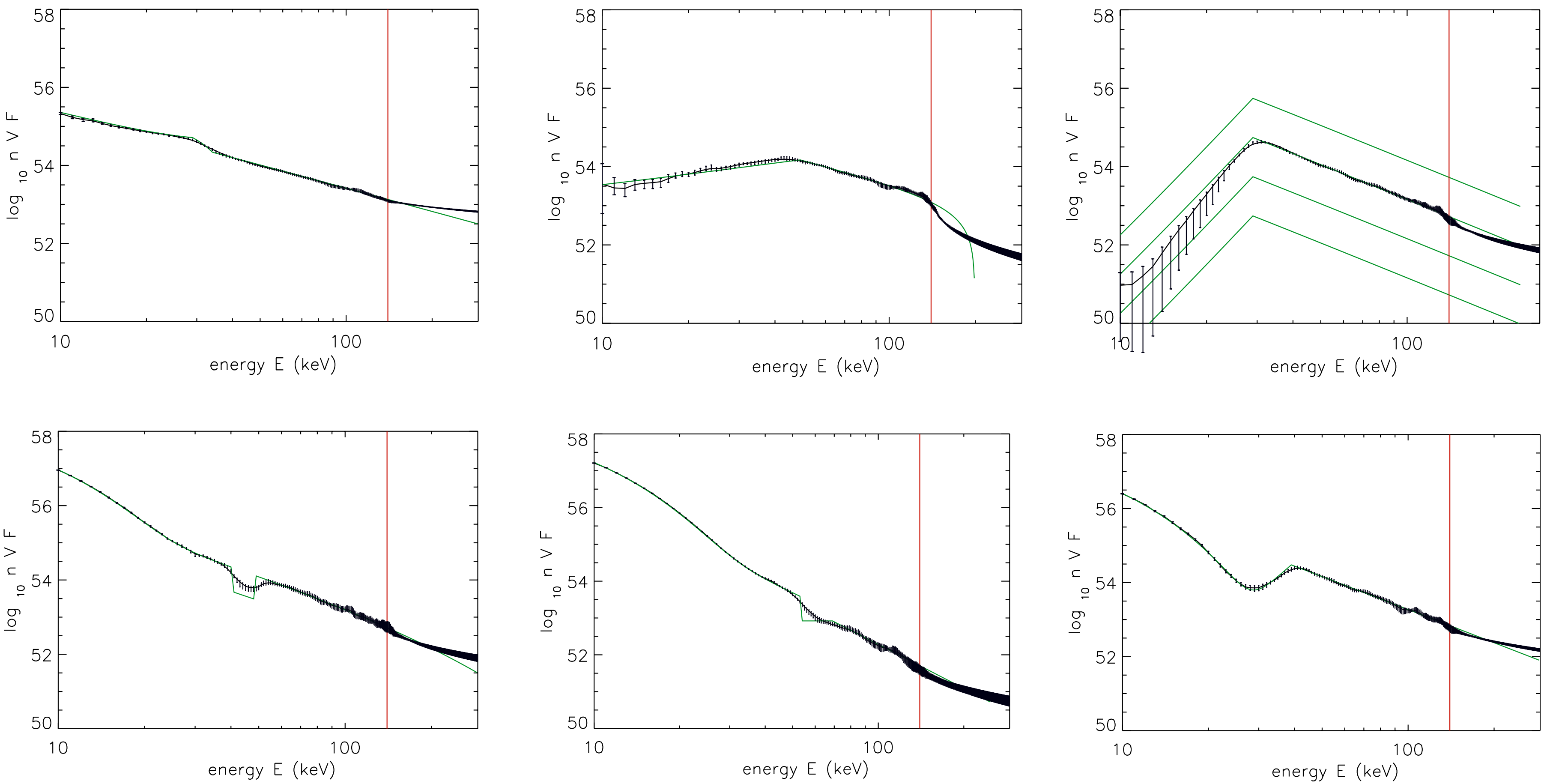
cumulative residuals: $$S^{(j)}(q) = \frac{1}{j} \sum_{i=1}^{j} r_i^{(k)}(q),$$

blind test - 1



Photon Spectra
CASE A
CASE B
CASE C
Photon Spectra
CASE D
CASE E
CASE F
I(ϵ) (cm⁻² s⁻¹ keV⁻¹)
ϵ (keV)
ϵ (keV)
Electron Spectra
CASE A
CASE B
CASE C
Electron Spectra
CASE D
CASE E
CASE F
nVF (10⁵⁰ cm⁻² s⁻¹ keV⁻¹)
E (keV)
E (keV)

blind test - 2


58
56
54
52
50
log 10 n V F
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100
energy E (keV)
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energy E (keV)
58
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52
50
log 10 n V F
10
100
energy E (keV)

uncertainties - 1

- forward-fit of photon spectra by means of parameterized models
- an MCMC method samples the parameter space to compute the posterior distribution
- detailed confidence intervals for the parameters

Ireland J, Tolbert A K, Schwartz R A, Holman G D and Dennis B R, ApJ, 2013

uncertainties 2

Bayesian Confidence Limits of electron spectra
(G. Emslie and A.M Massone)

- The expectation values of the source brightness and its variance in a given photon energy bin are in general not equal to the number n of counts observed in that energy bin but depend on n and on prior knowledge of the overall photon spectrum

- Using Bayesian analysis we computed the posterior probability for an event with n* counts given an event with n counts and with a given expectation values

- We used this result to determine the confidence strip in a mean electron flux spectrum computed by means of regularization

the ion problem -1

$$I(\varepsilon) = \sum_k \sum_j \int_0^\infty F_j(E) A_k j_{kj}(\varepsilon, E) dE$$

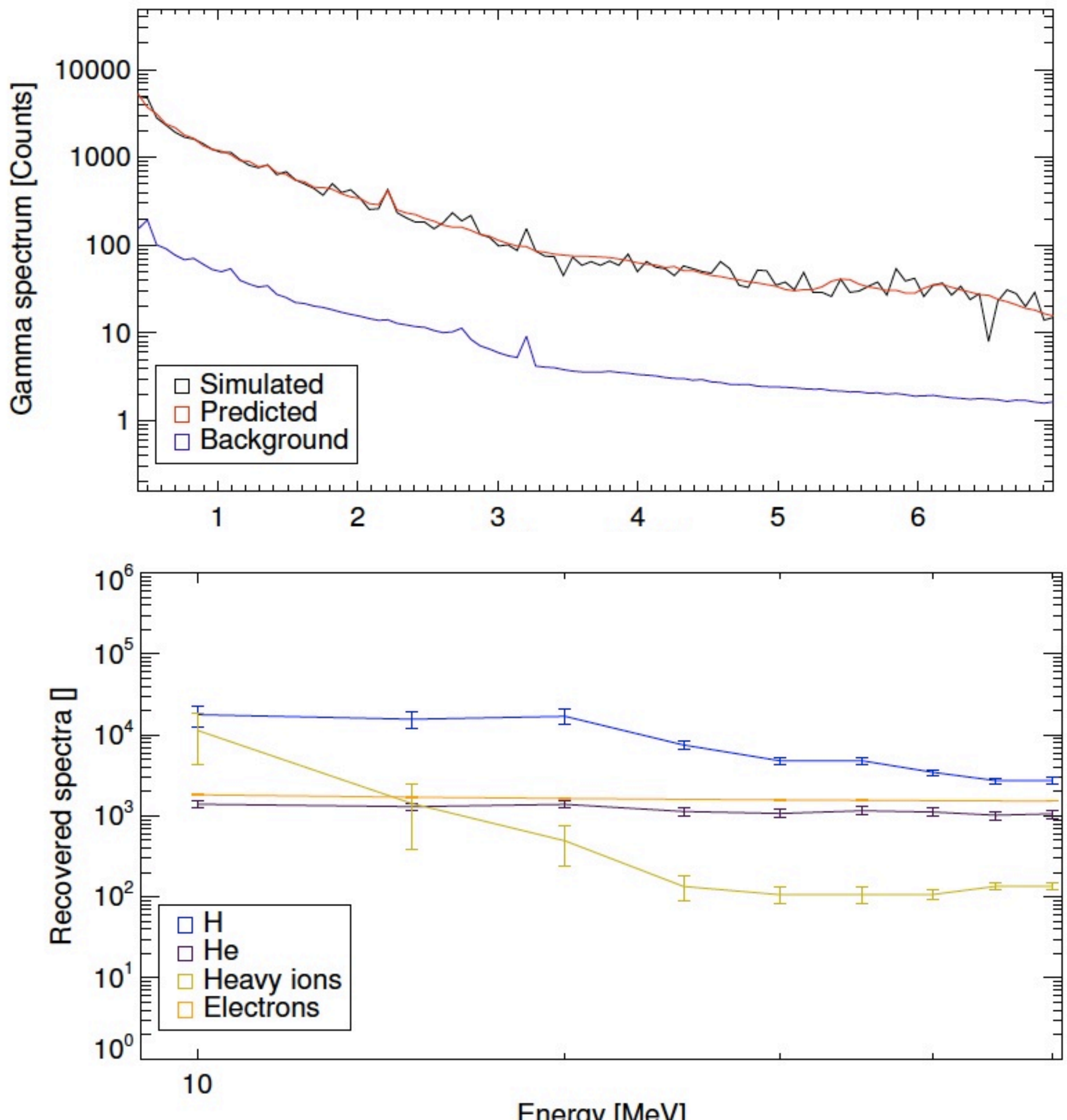
$$y = aHx$$

find (a, x) such that: $KL(x, a), y) = \min$

$$x_j \leftarrow x_j \frac{1}{\sum_{i,p} a_p H^p_{ij}} \sum_{i,p} a_p H^p_{ij} \frac{y_i}{\sum_{q,p} a_p H^p_{iq} x_q} \; ,$$

$$a_p \leftarrow a_p \frac{1}{\sum_{i,j} H^p_{ij} x_j} \sum_{i,j} H^p_{ij} x_j \frac{y_i}{\sum_{q,p} a_p H^p_{iq} x_q} \; .$$

the ion problem - 2



the ion problem -3

target

Recovered	Type
1.0 ± 0.0	H
0.0 ± 0.0	He3
0.000093 ± 0.000009	He
0.00006 ± 0.00008	C
0.000008 ± 0.000004	N
0.00011 ± 0.00012	O
0.00003 ± 0.00002	Ne
0.00055 ± 0.00018	Mg
0.0003 ± 0.0007	Al
0.00019 ± 0.00010	Si
0.00185 ± 0.00012	S
$9.0 \times 10^{-12} \pm 2.9 \times 10^{-8}$	Ca
0.000016 ± 0.000004	Fe

accelerated particles

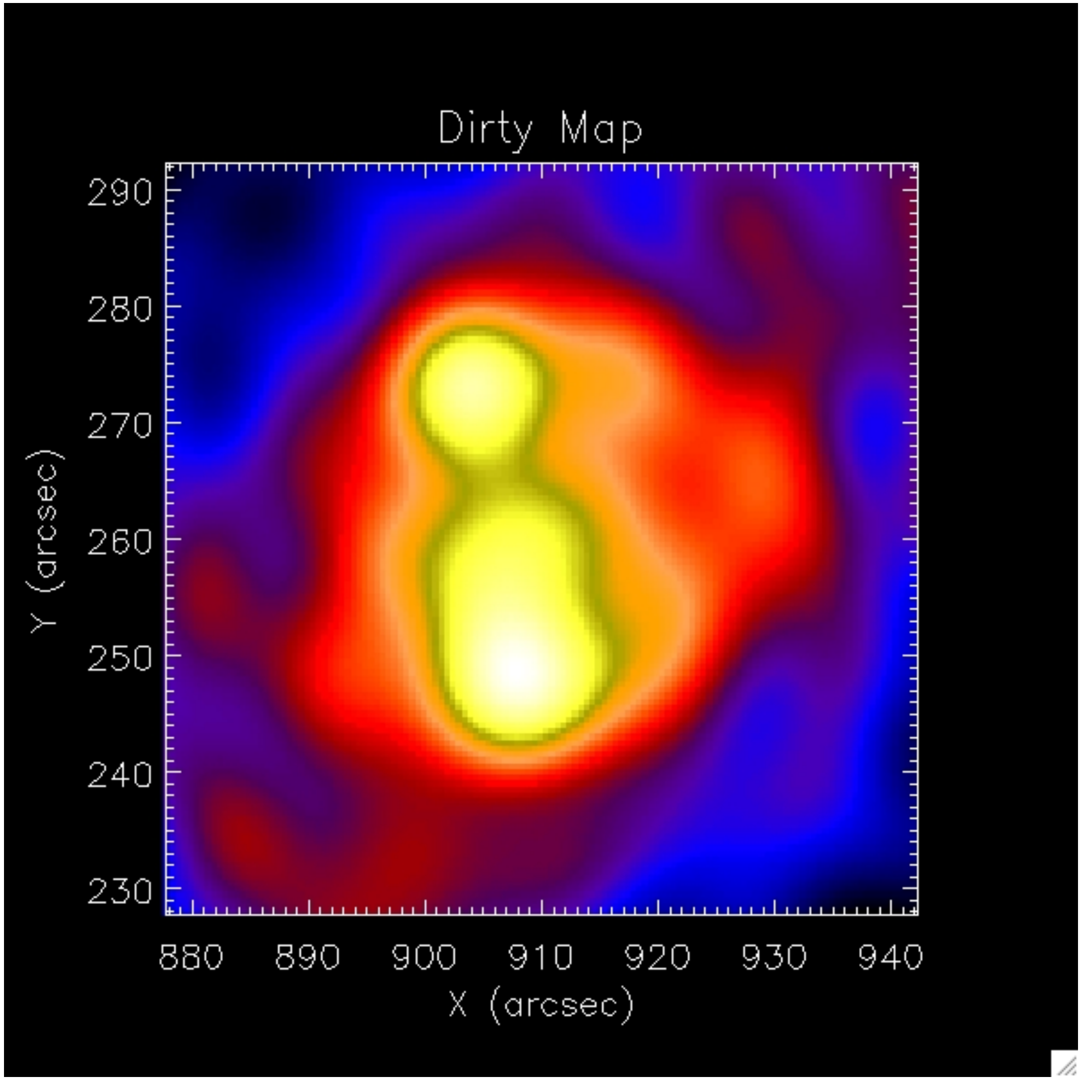
Recovered	Type
1.0 ± 0.4	H
0.14 ± 0.04	He
0.010 ± 0.005	C
0.012 ± 0.005	N
1.8 ± 0.7	O
0.011 ± 0.006	Ne
0.0026 ± 0.0010	Mg
0.024 ± 0.018	Al
0.025 ± 0.011	Si
0.015 ± 0.007	S
0.00003 ± 0.00009	Ca
0.011 ± 0.004	Fe

imaging

Back-Projection from visibilities instrument-independent

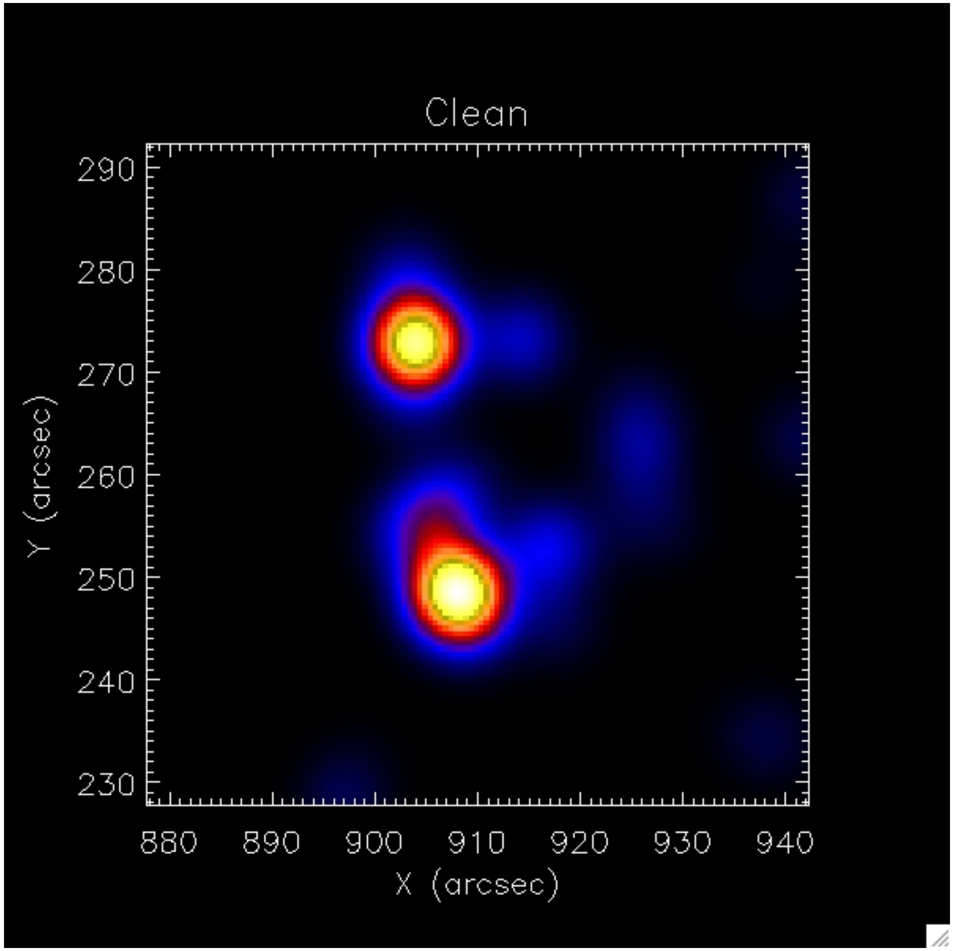
vis_bpmap

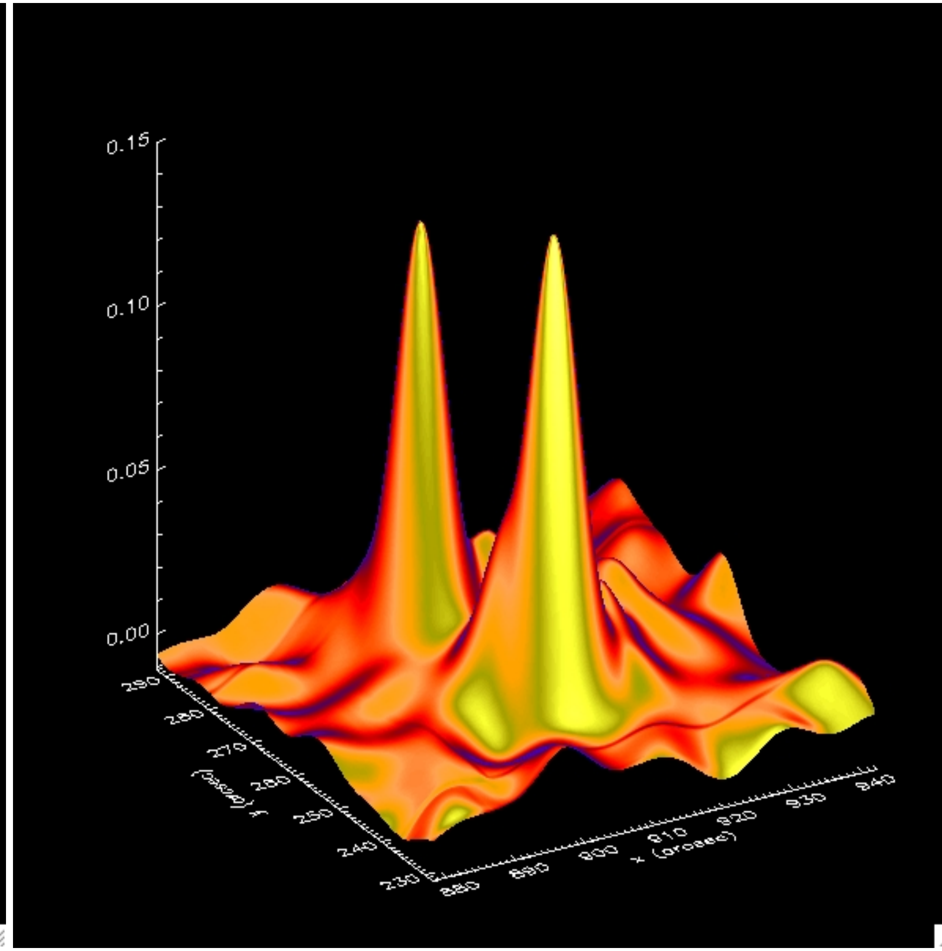
20 February 2002
(11:06:02 - 11:06:34 UT)



Clean algorithm from visibilities: vis_clean

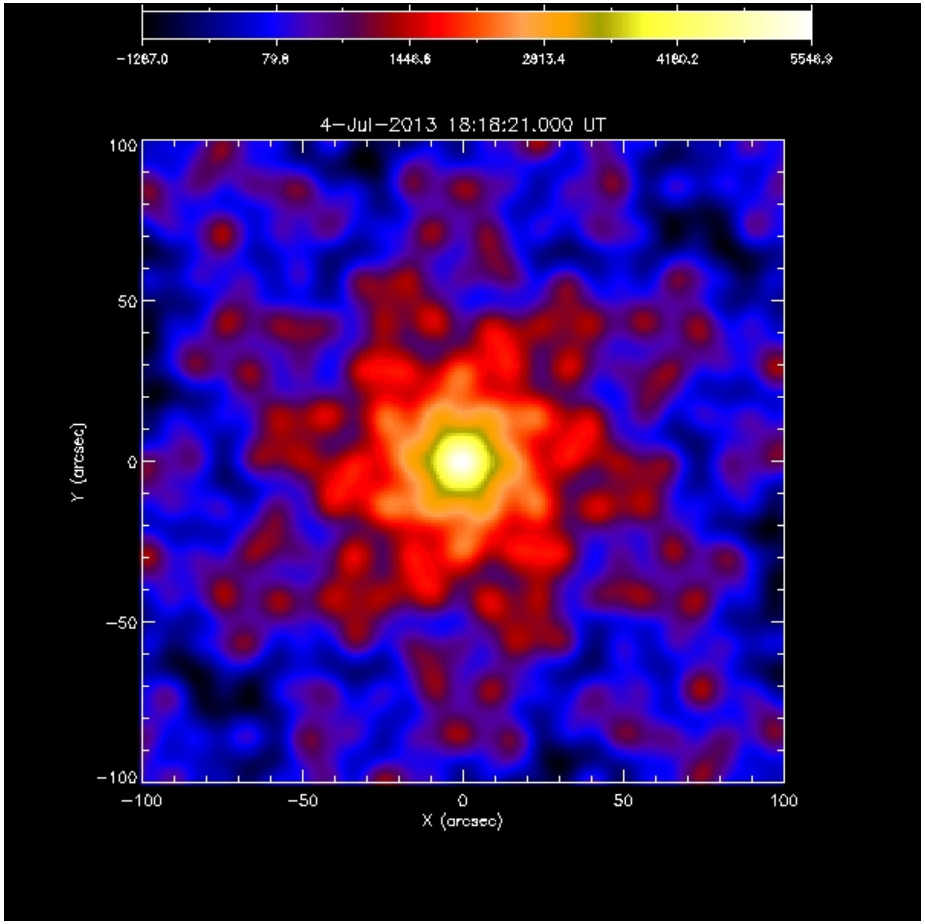
HESPE developed a new version of the CLEAN algorithm where inputs are visibilities instead of counts

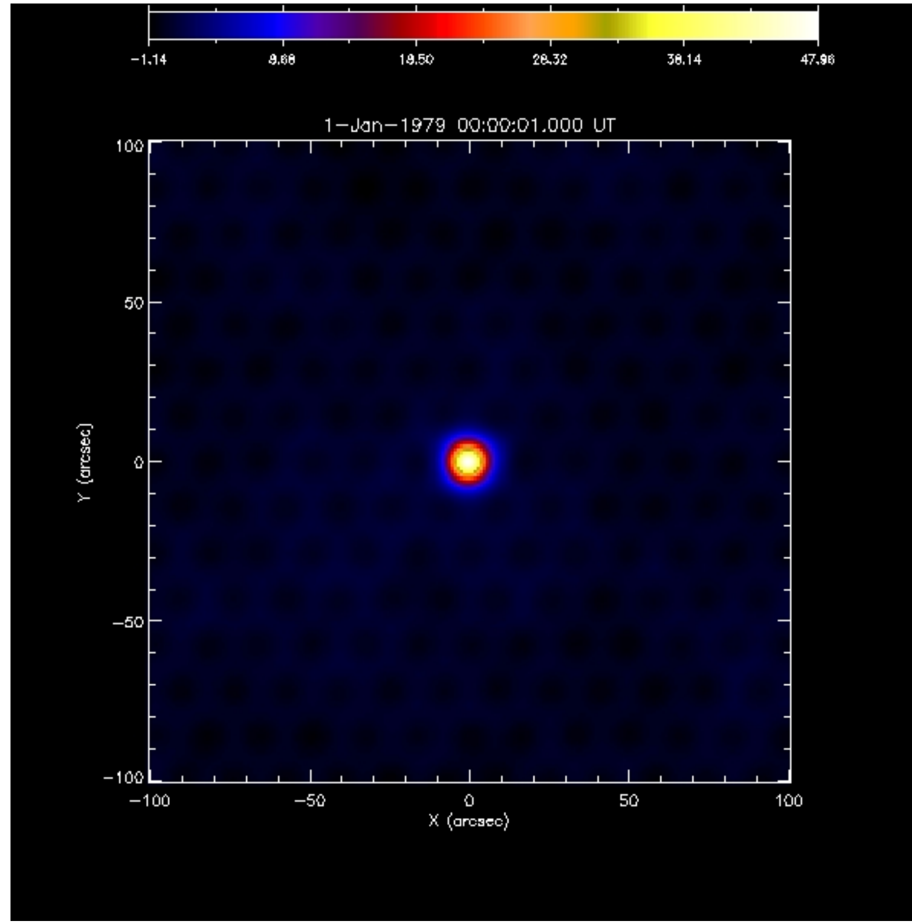




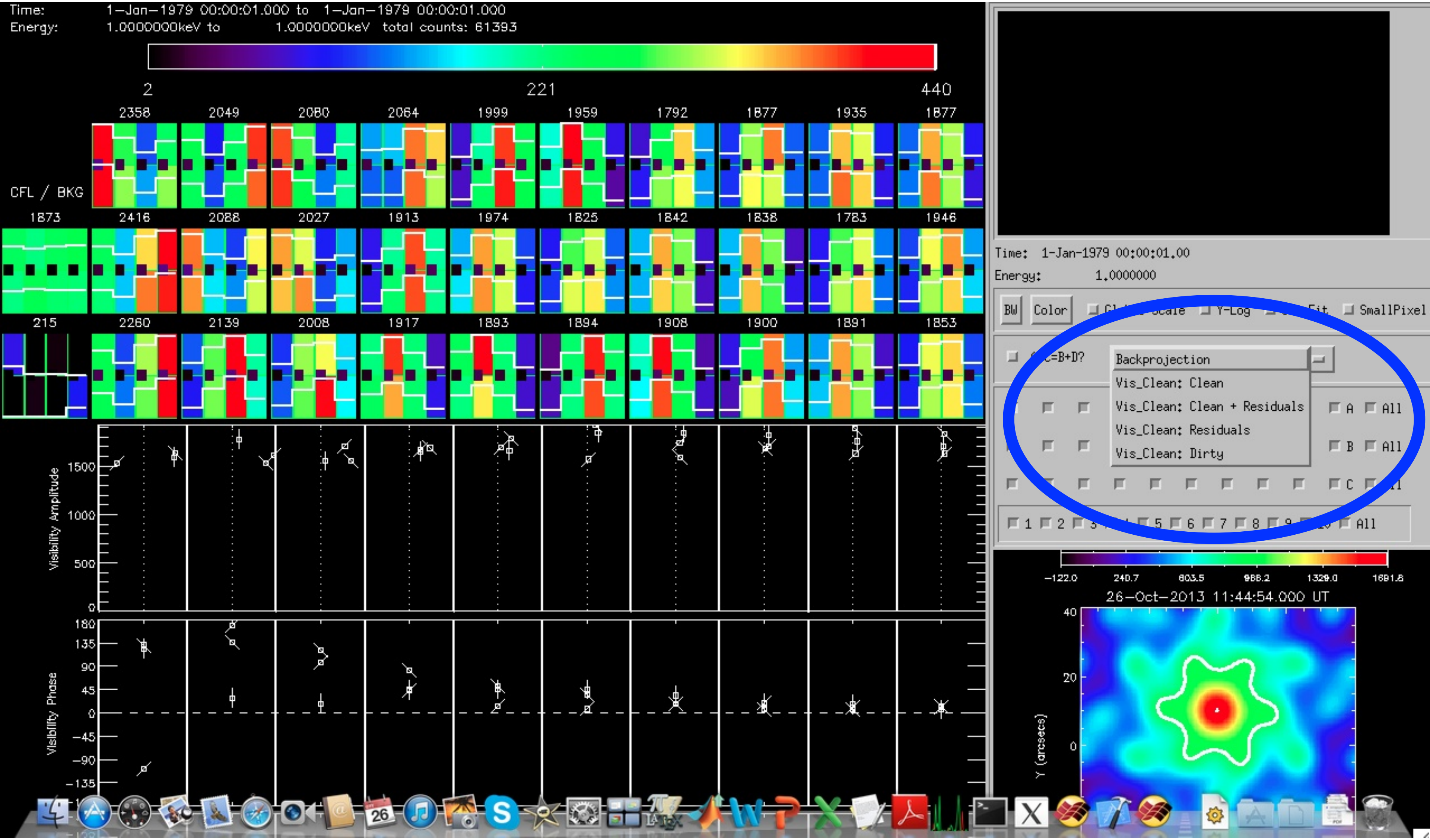
February 20, 2002 11:06:10 - 11:06:40 UT

Clean algorithm from visibilities: instrument independent





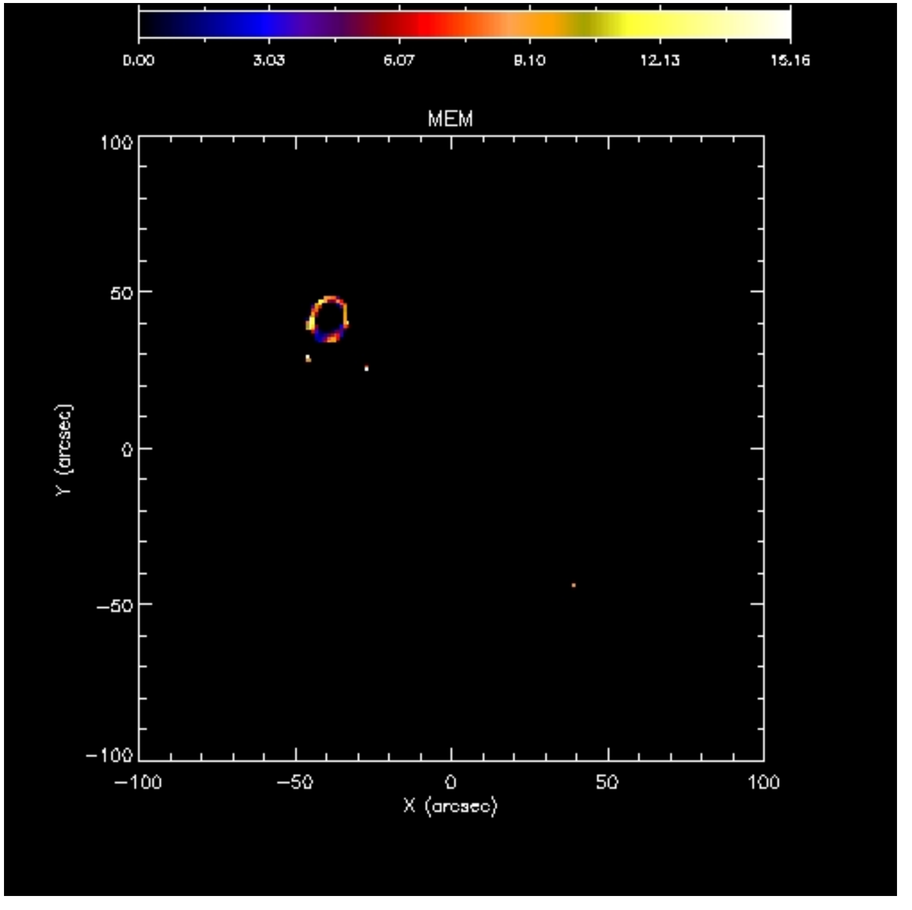
Clean & backprojection algorithms in STIX Software



MEM_NJIT

The maximum entropy routine is not always successful:

sometimes MEM–NJIT iterates too much



MEM_NJIT

MEM objective function $$J = H - \alpha\chi^2 - \beta F$$

where $$H = -\sum_j T_j \ln\left(\frac{T_j}{m_j e}\right)$$

$$\chi^2 = \sum_i \frac{|V_i - V_i'|}{\sigma_i^2} - n_v$$

$$F = \sum_j T_j - F'$$

T = actual reconstruction

V = visibilities predicted by T

V' = observed visibilities

F' = observed flux

m = guess map

n_v = number of visibilities

$$\varepsilon \approx 0.03 \qquad \xi_j = \frac{T_j}{m_j} - \frac{m_j}{T_j}$$

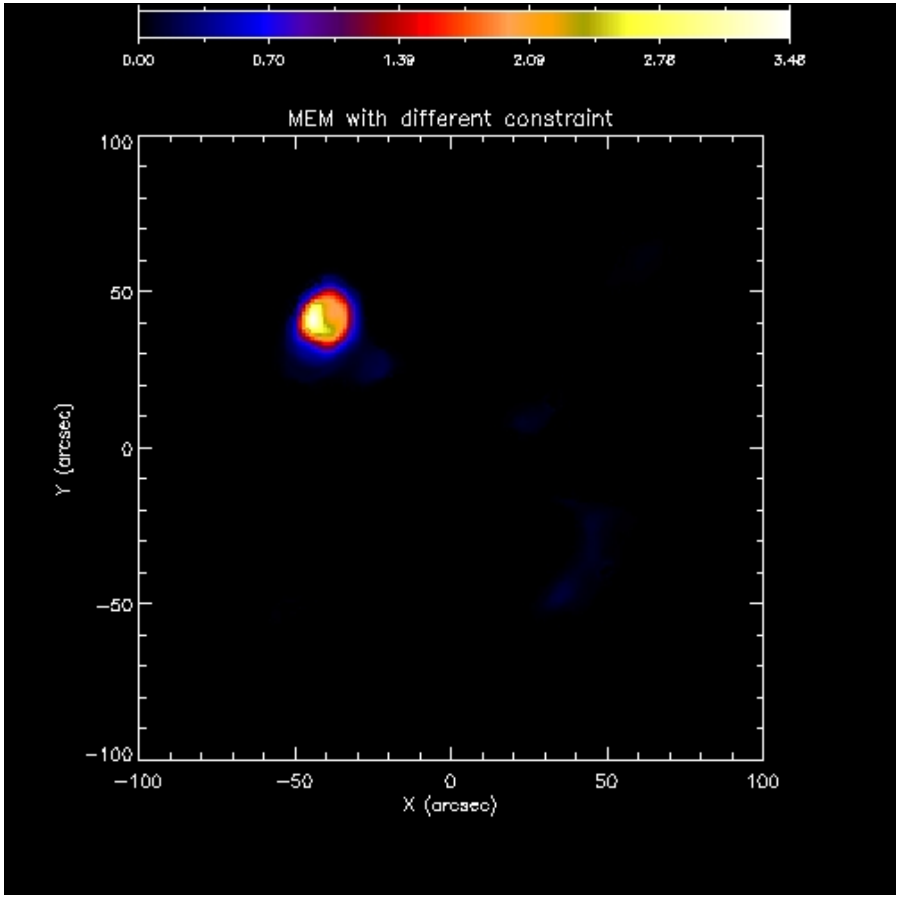
under the constraints:

$$|\chi^2| \leq \max(\varepsilon n_v, \sqrt{2n_v}, |\nabla_\xi \chi^2| \sqrt{\sum_j (\delta\xi_j)^2})$$

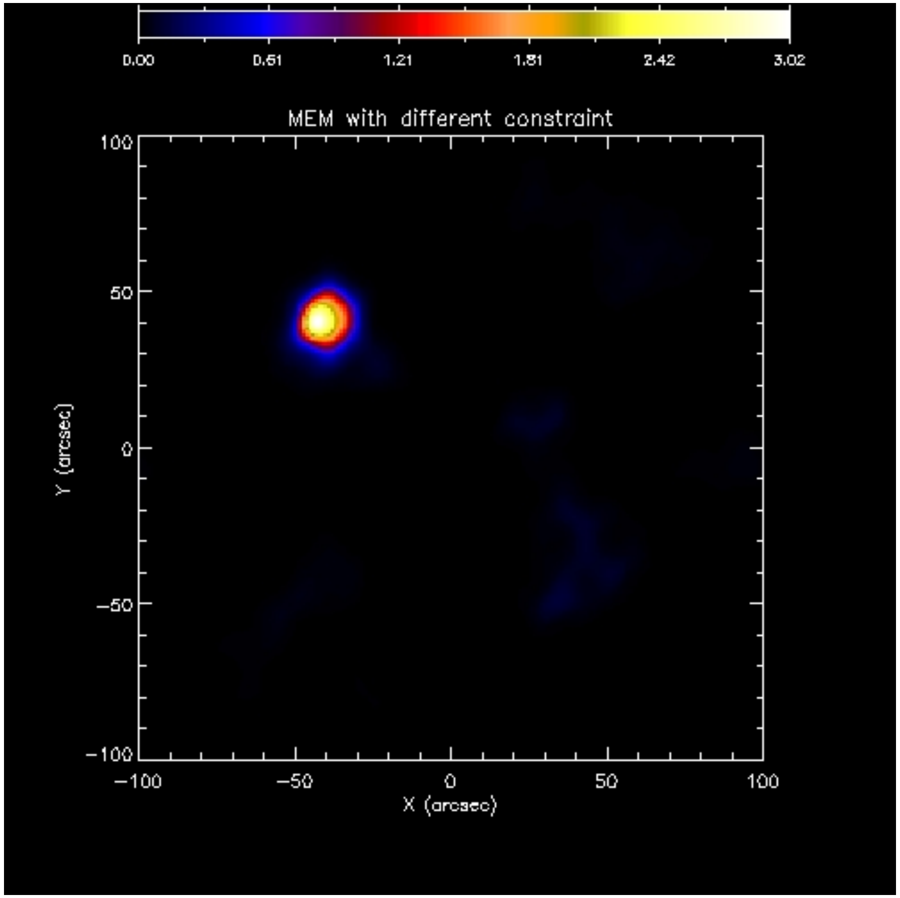
$$|F| \leq \max(\varepsilon n_v, \sqrt{2n_v}, |\nabla_\xi F| \sqrt{\sum_j (\delta\xi_j)^2})$$

MEM_NJIT

Just one constraint verified:



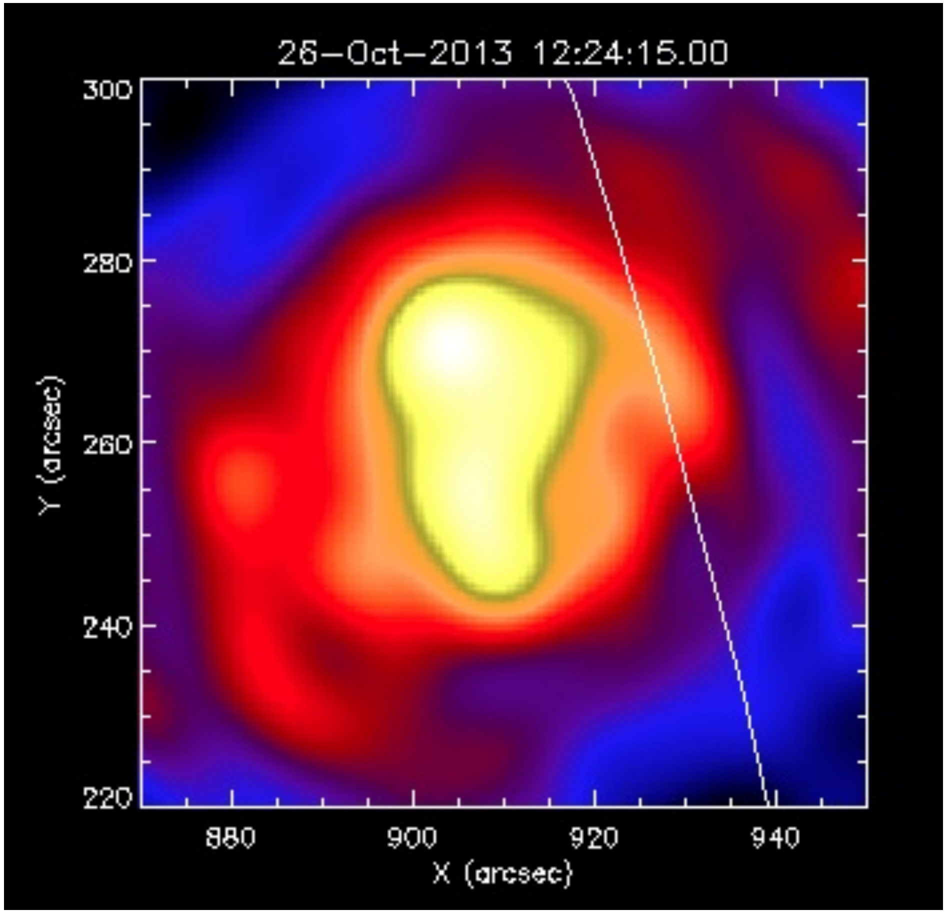
The two constraints relaxed:



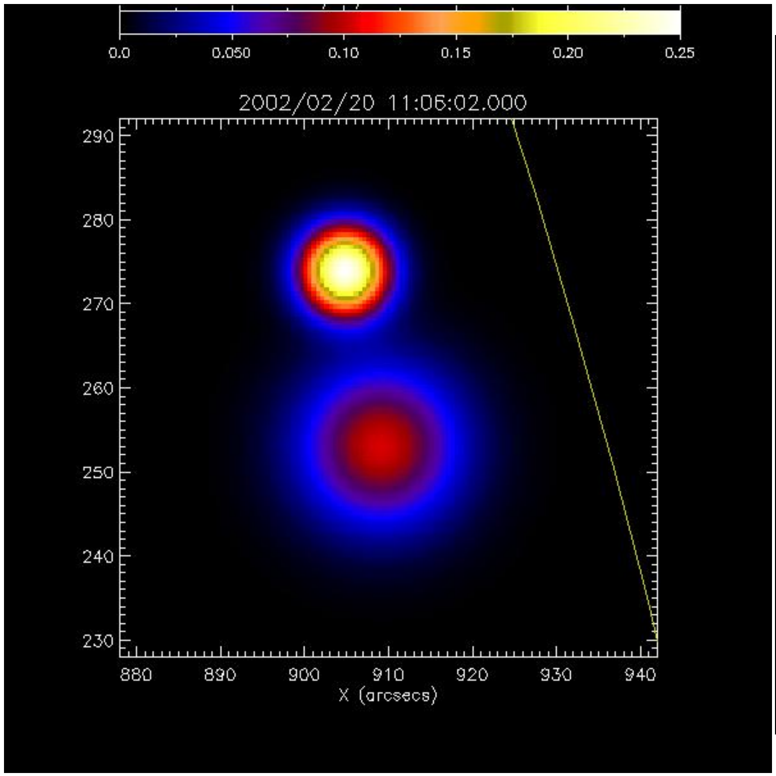
Vis Forward Fit: instrument independent

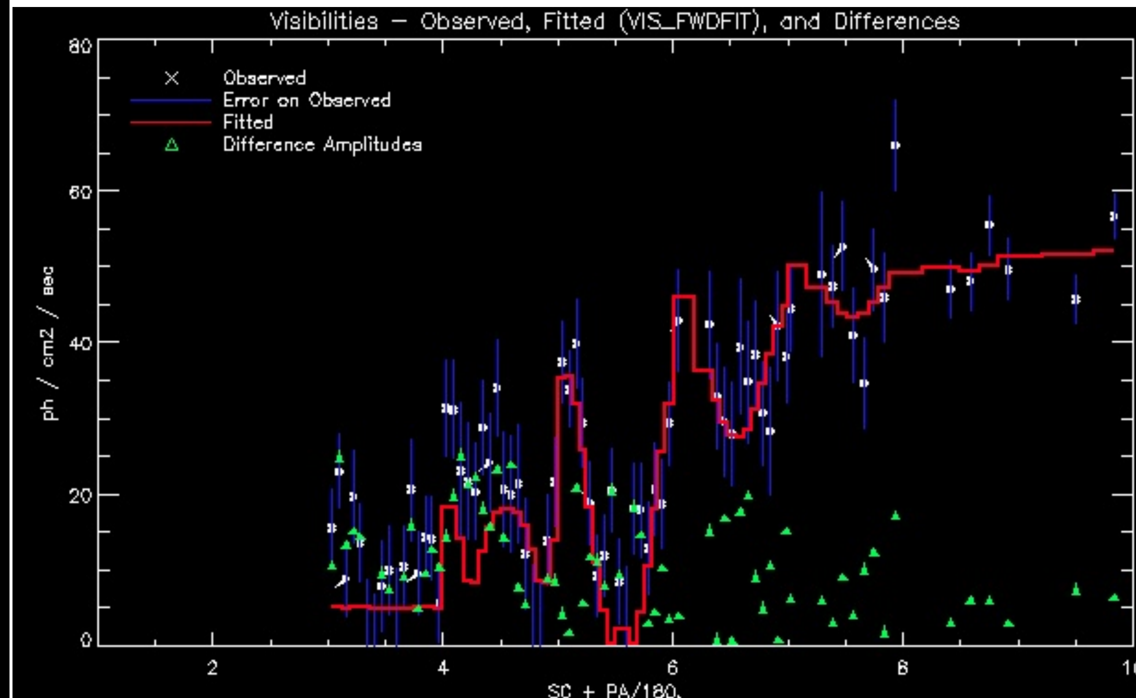
RHESSI data:

20 February 2002
(11:06:02 - 11:06:34 UT)
18-22 keV



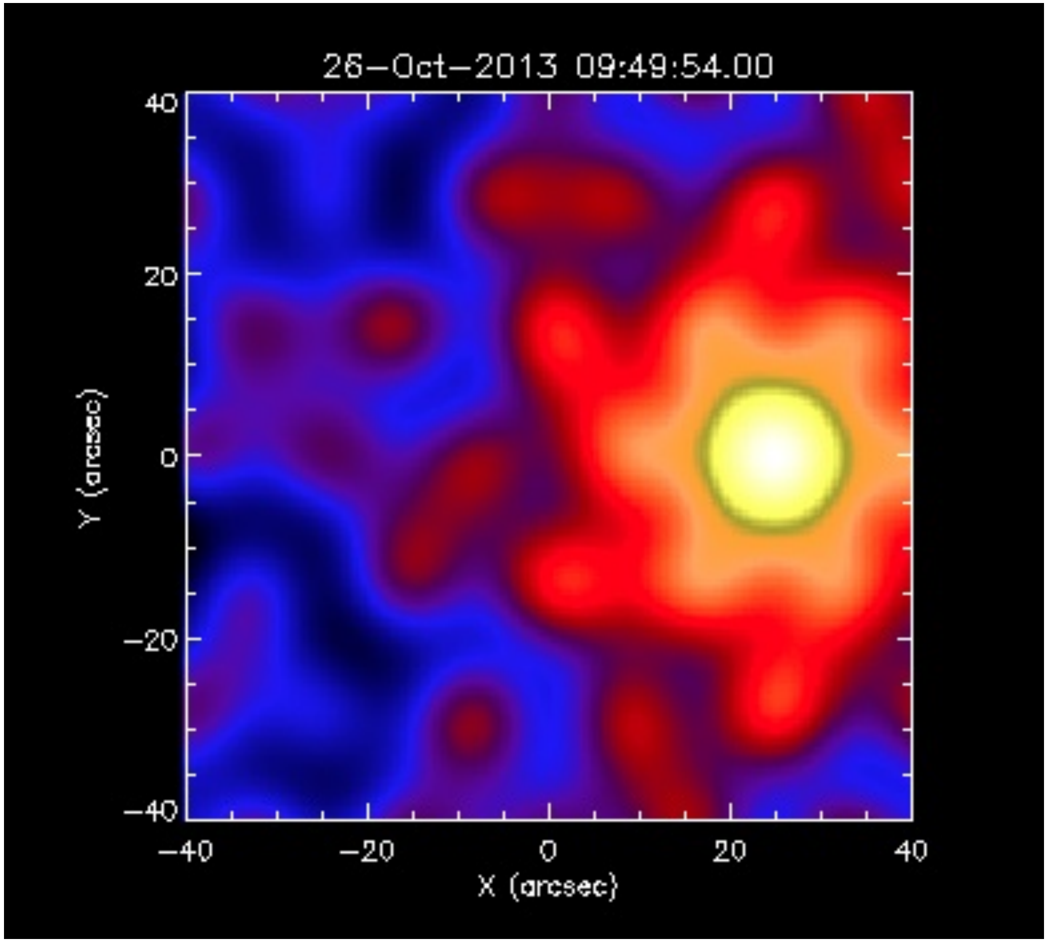
Vis Forward Fit: instrument independent



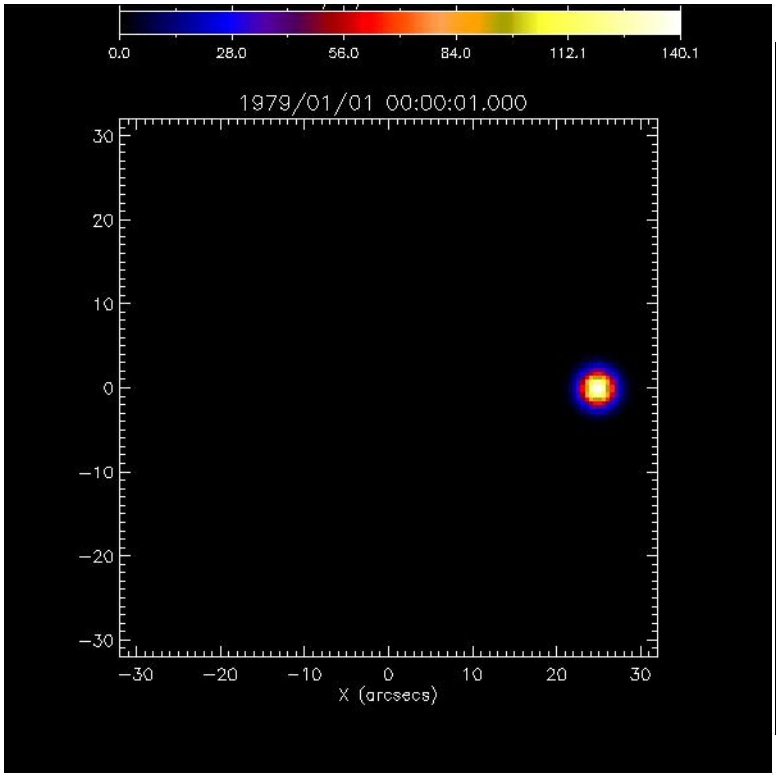


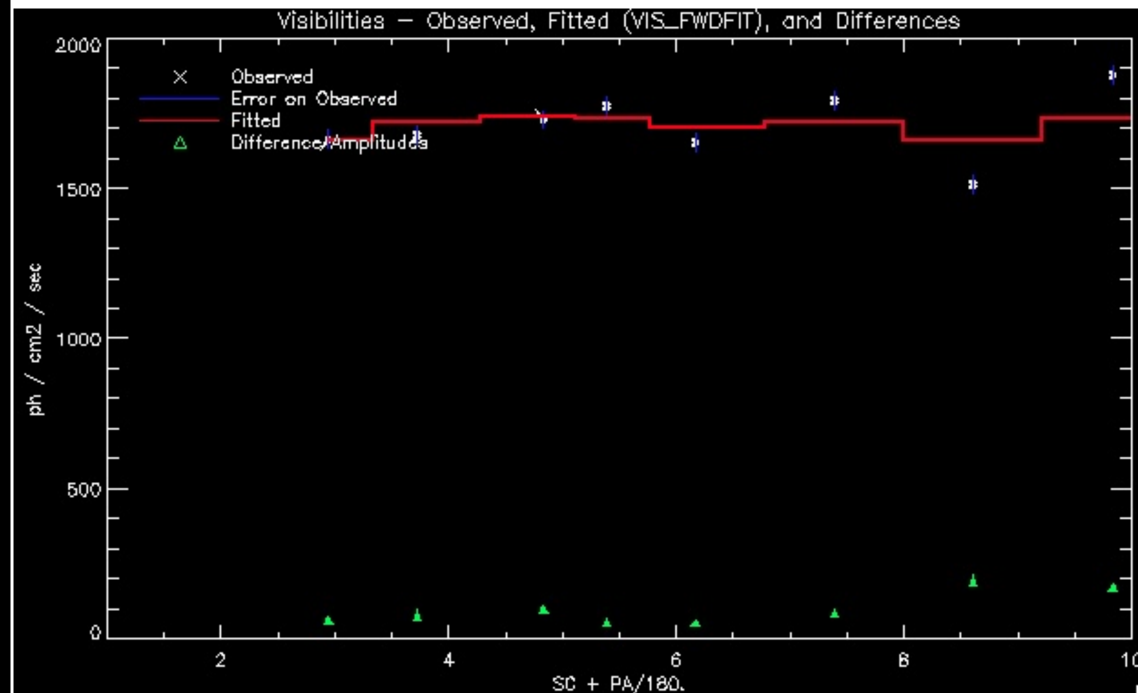
Vis Forward Fit: instrument independent

STIX Simulation



Vis Forward Fit: instrument independent





Imaging test

RHESSI IX --- Genova (*first* test design)
RHESSI X --- Annapolis (Sam analysis)
RHESSI XI --- Glasgow (Anna analysis)
RHESSI XII --- Nanjing (results collected)

Algorithms:

MEM_NJIT & uv_smooth (A.M. Massone)
VIS FORWARD_FIT (M. Battaglia)
Pixon (R. Schwartz)
Clean (A. Gopie)

Making images

For each algorithm and for each count rate:

Default image with subcollimators 1-9, 2-9, 3-9

Best fine-tuned image with subcollimators 1-9, 2-9, 3-9

Analysis on about 900 reconstructed maps:

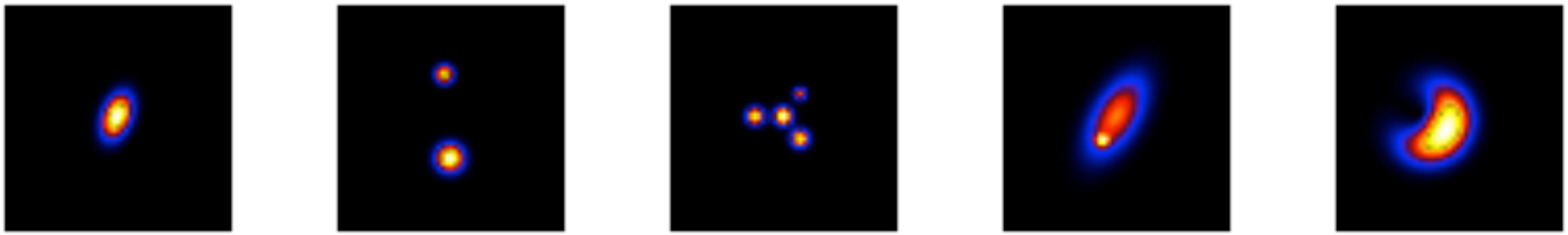
Images 1-> 5 (S. Krucker)
Images 6->10 (A.M. Massone)

- application of routines for the quantitative assessment of the algorithms’ performances

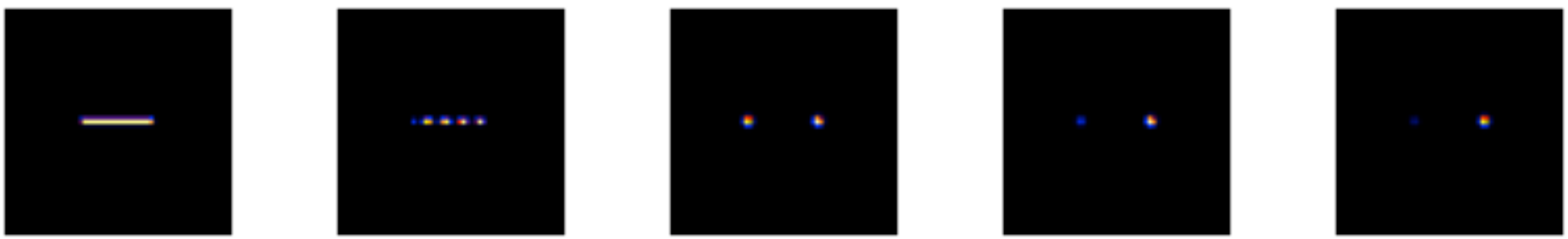
Test images

May 21, 2004	Feb 20, 2002	Jul 23, 2002	Dec 02, 2003	Aug 23, 2005
Single compact source, slightly elongated	Two footpoints (different size and intensity)	4 compact sources	Extended + compact	loop

SAM

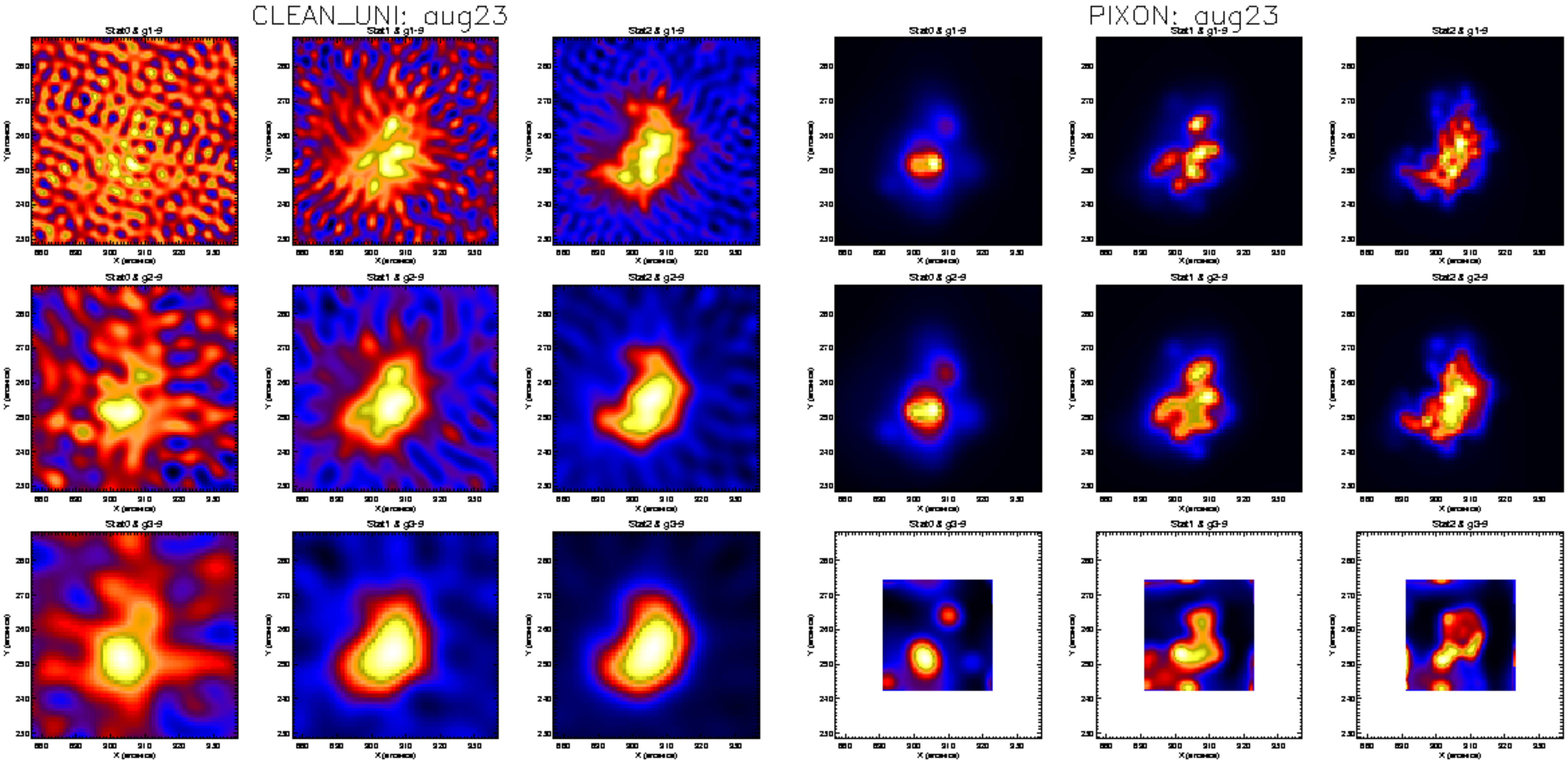


ANNA

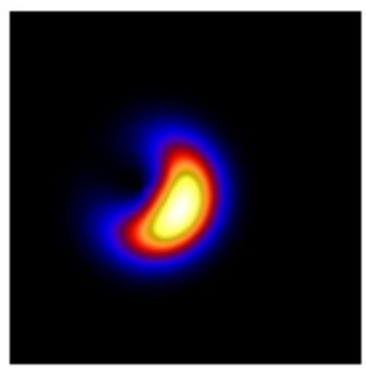


Line source
(15 arcsec x 1 arcsec)
Constant intensity   Varying intensity

Dynamic range test:
two point sources
(2 arcsec size each separated by 20 arcsec)
Flux ratio
1   5   10

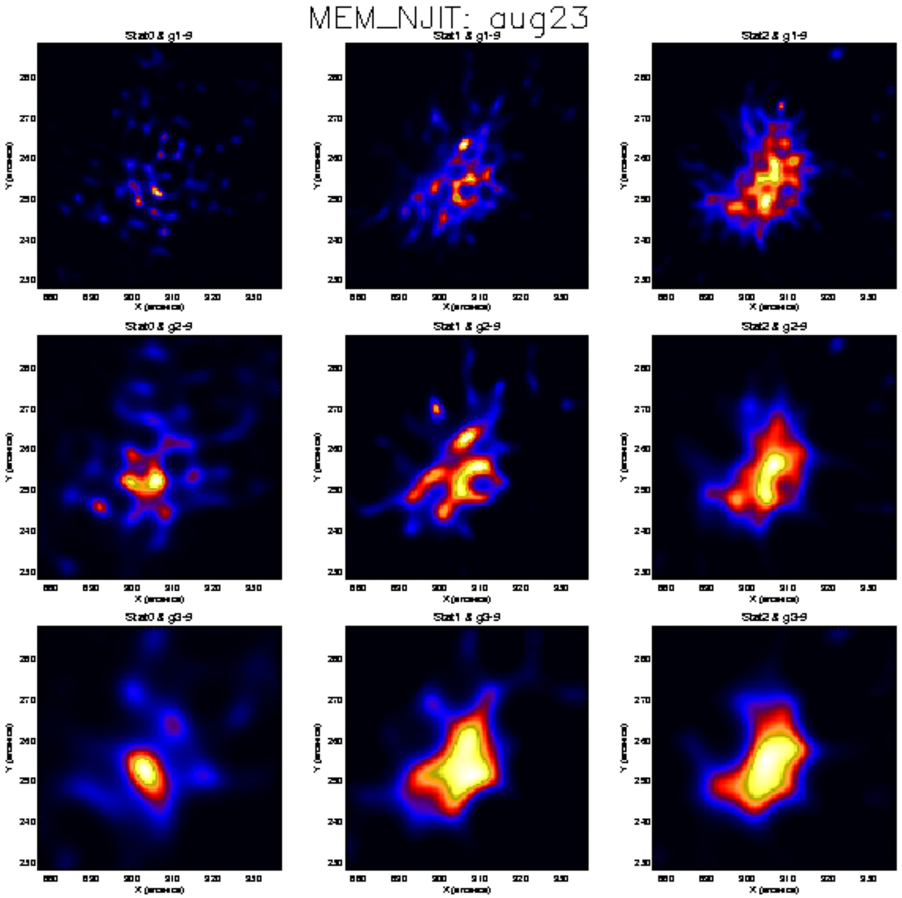


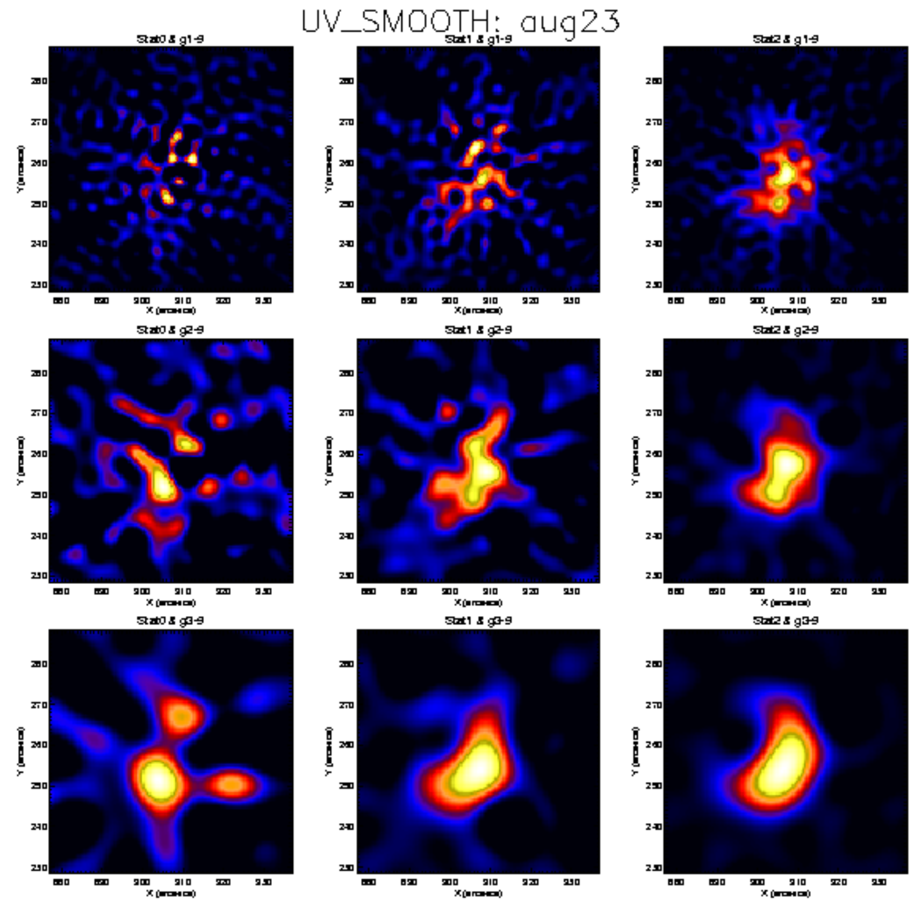
CLEAN



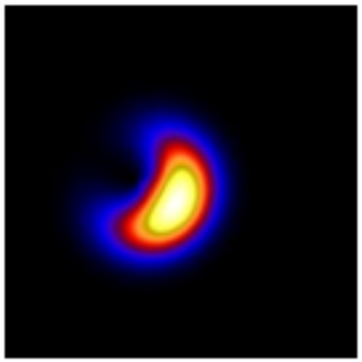
PIXON

Sam analysis: Aug 23



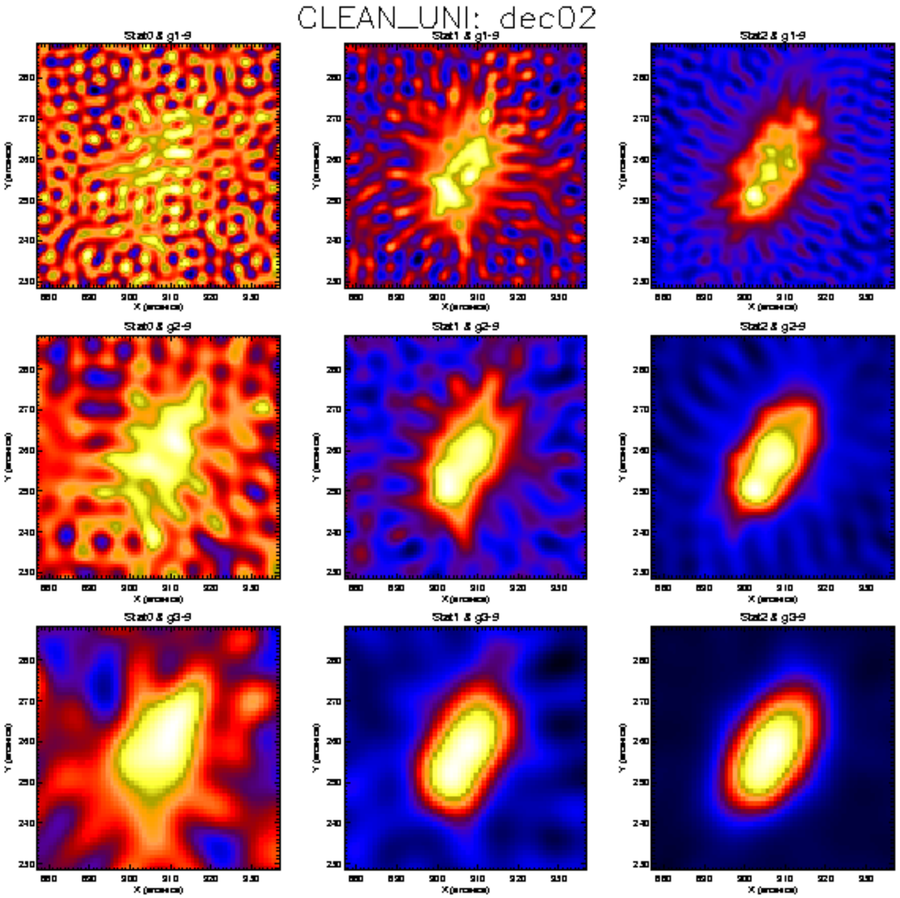


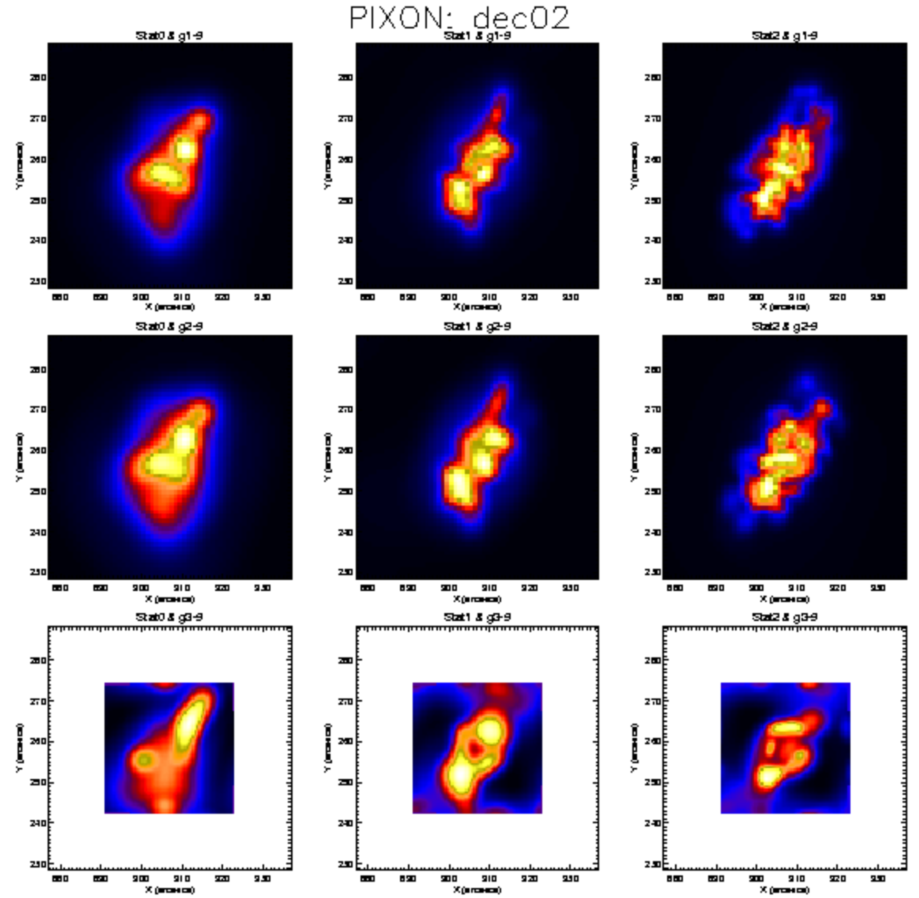
MEM_NJIT



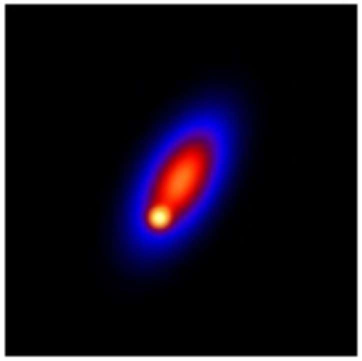
UV_SMOOTH

Sam analysis: Aug 23



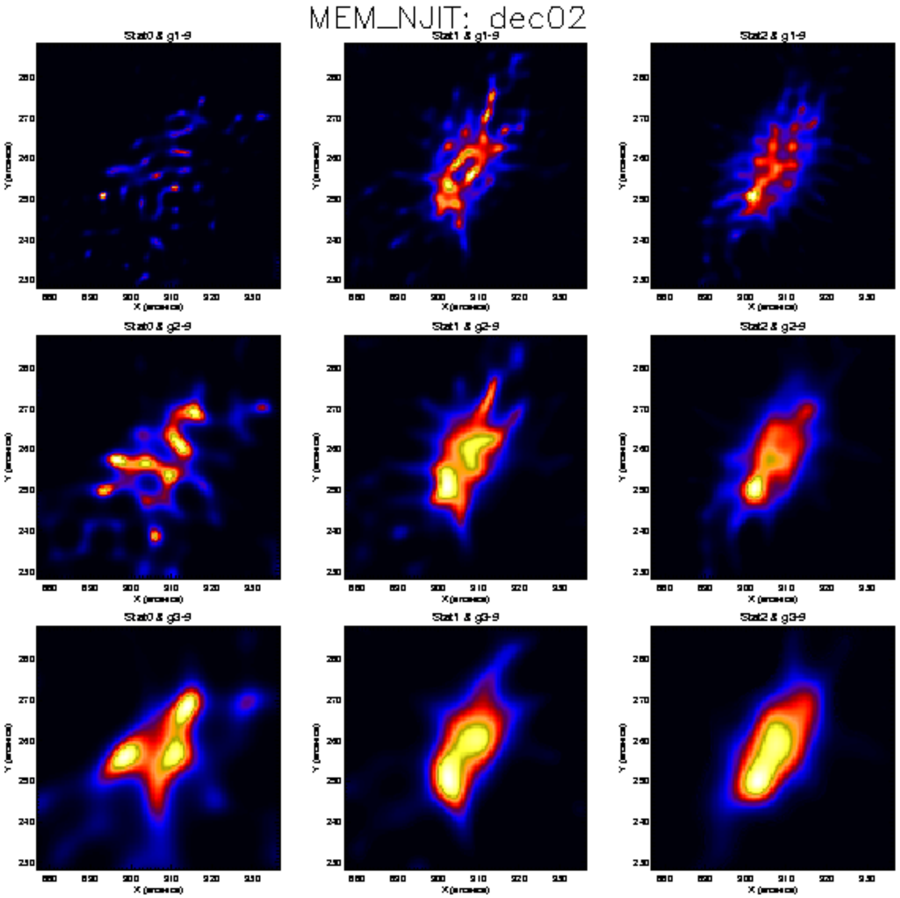


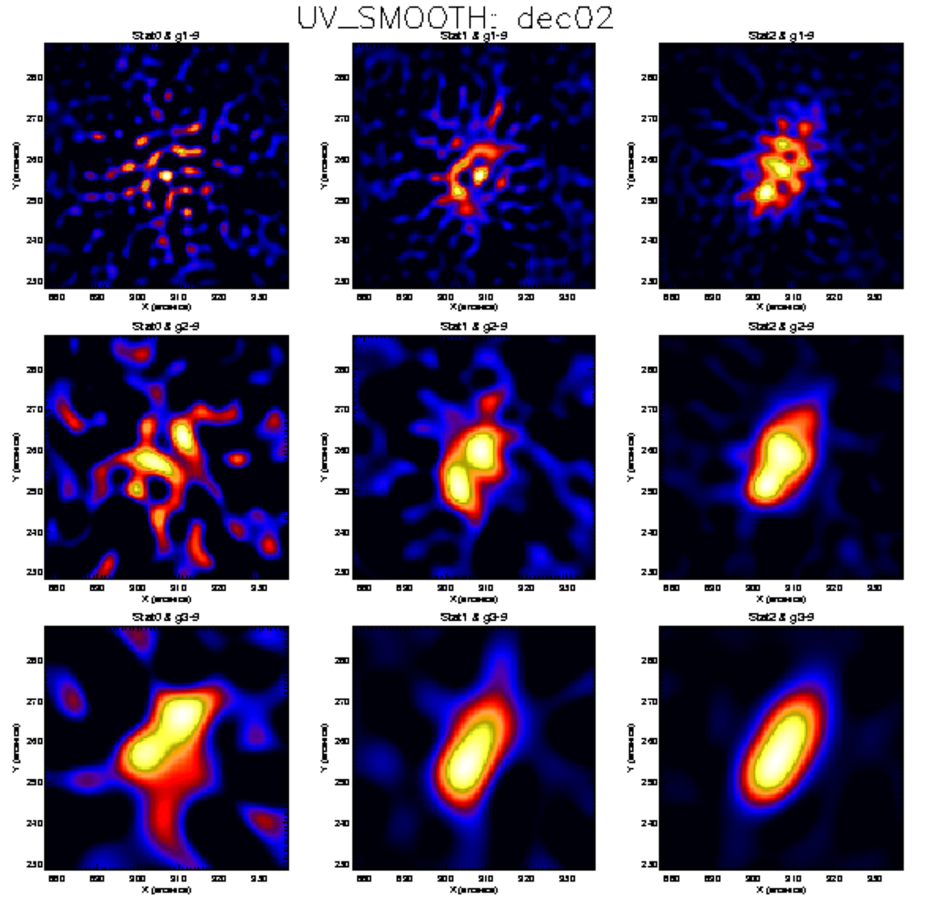
CLEAN



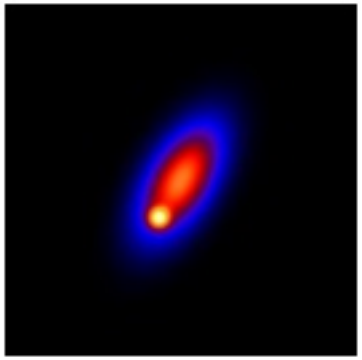
PIXON

Sam analysis: Dec 02



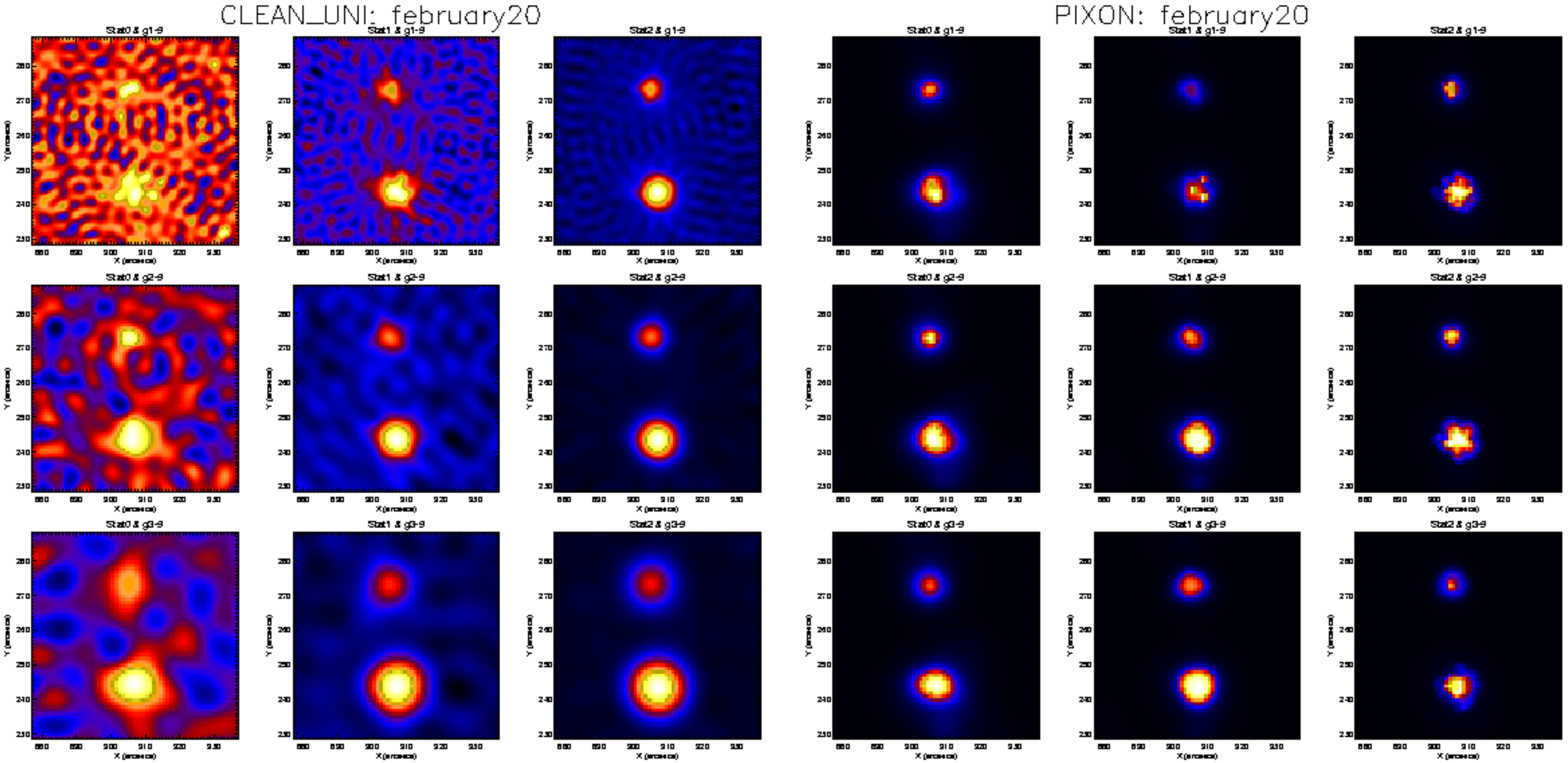


MEM_NJIT

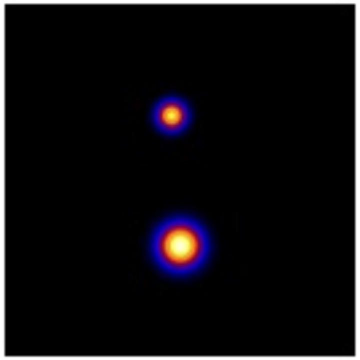


UV_SMOOTH

Sam analysis: Dec 02

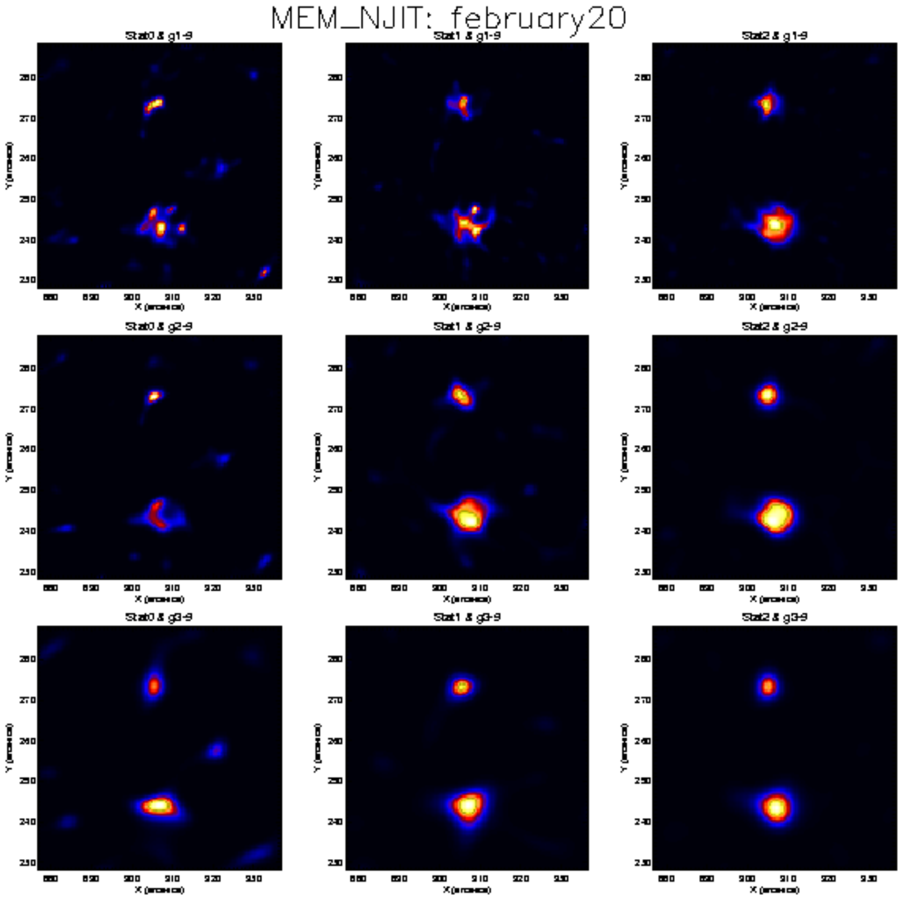


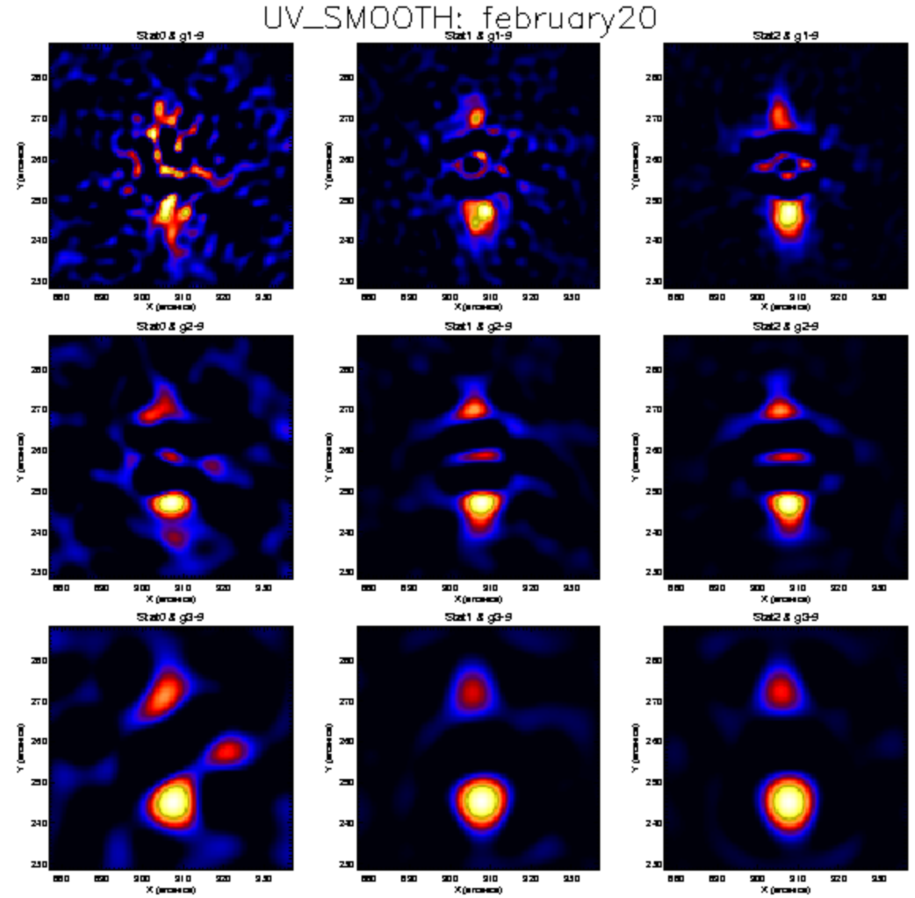
CLEAN



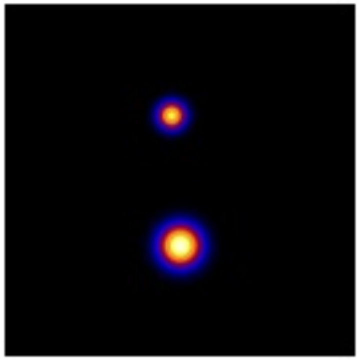
PIXON

Sam analysis: Feb 20



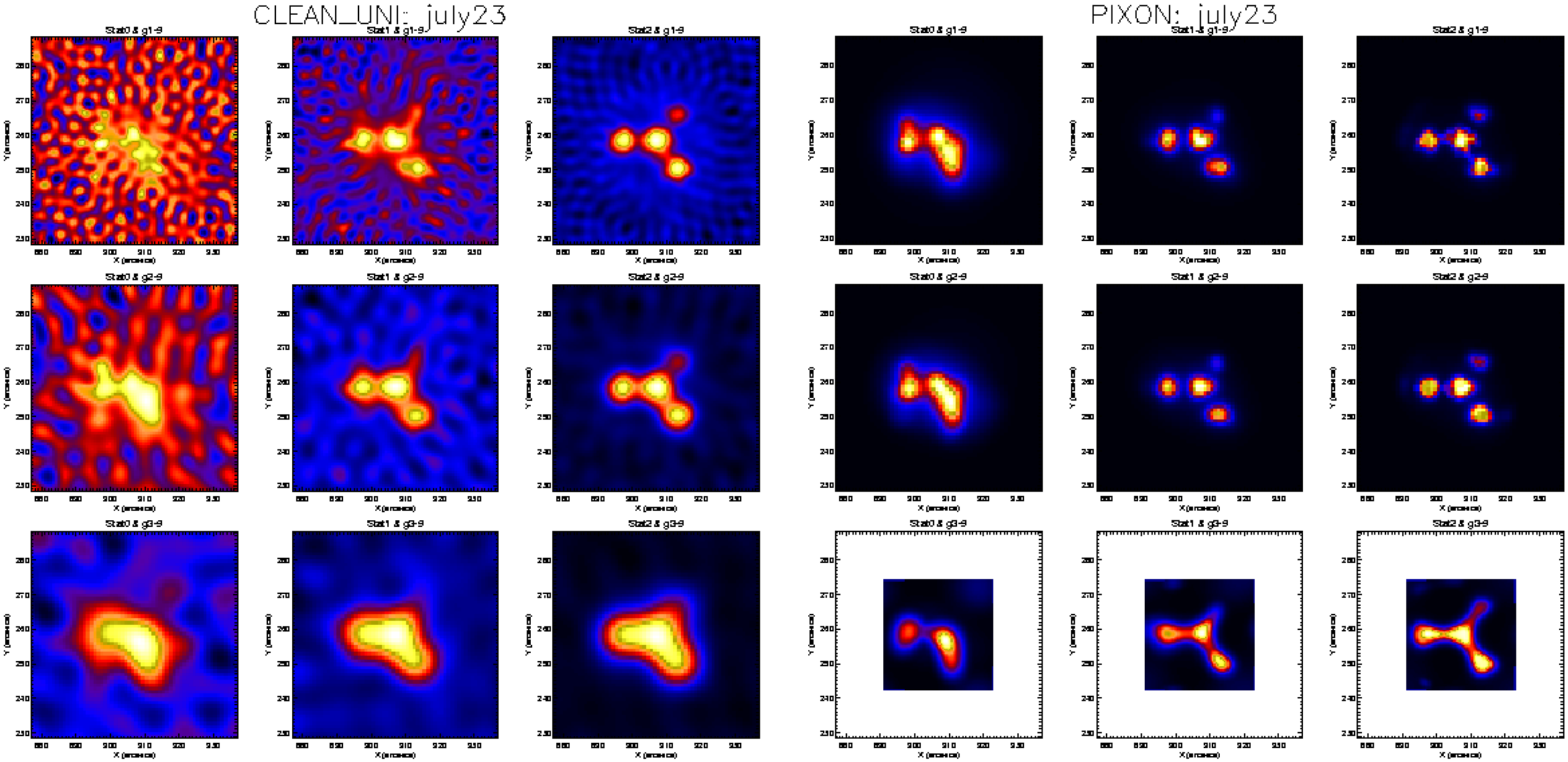


MEM_NJIT

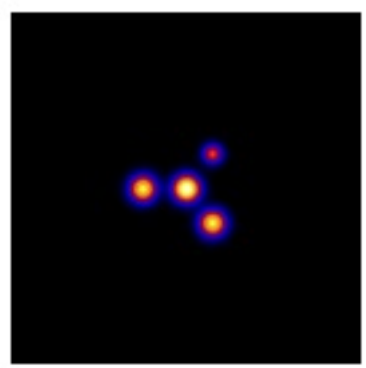


UV_SMOOTH

Sam analysis: Feb 20

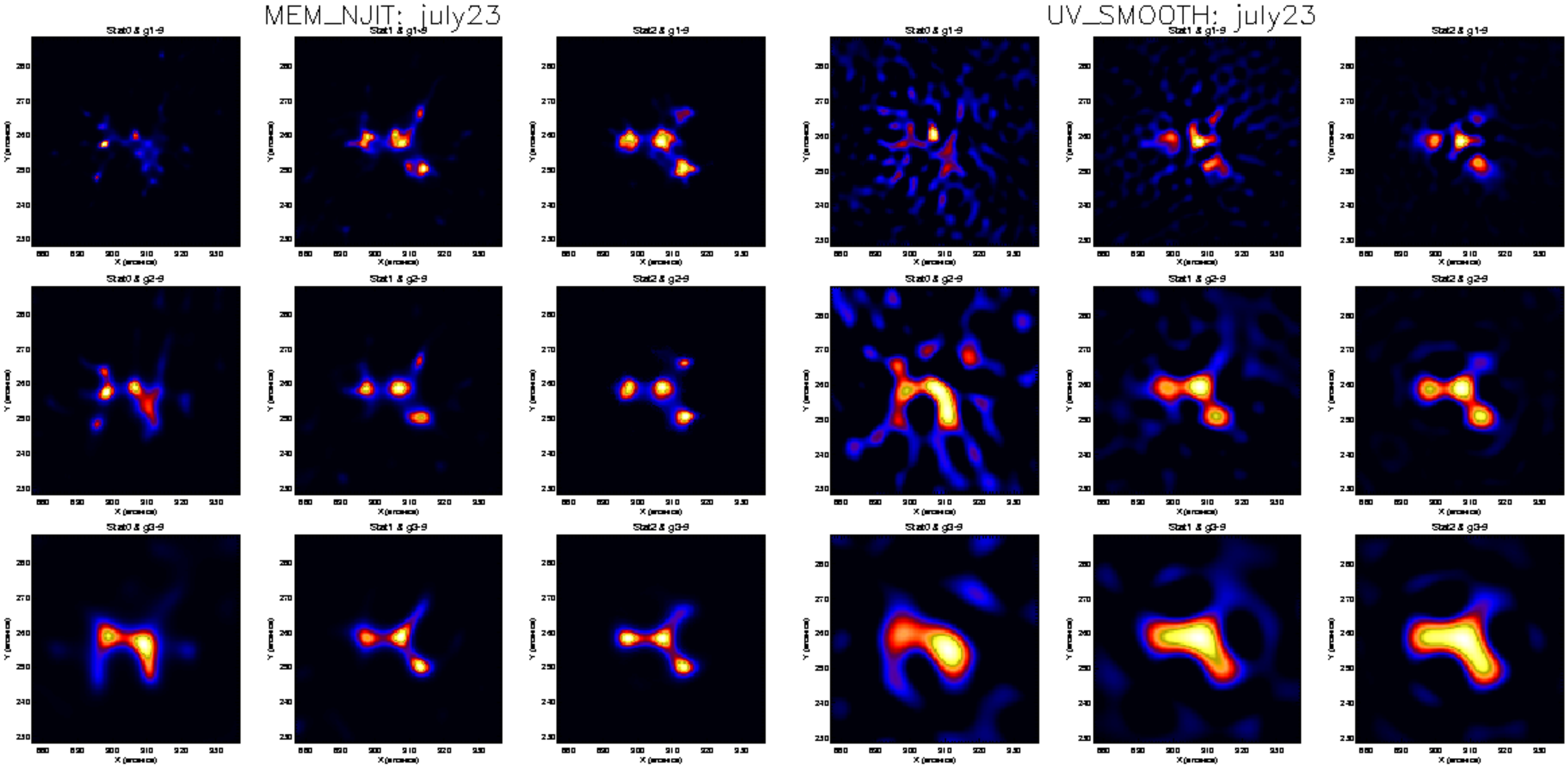


CLEAN

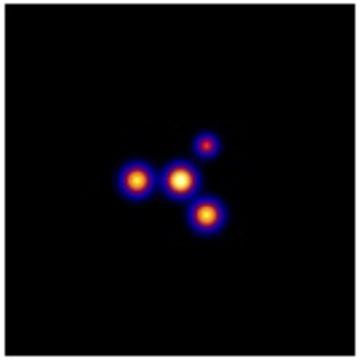


PIXON

Sam analysis: Jul 23

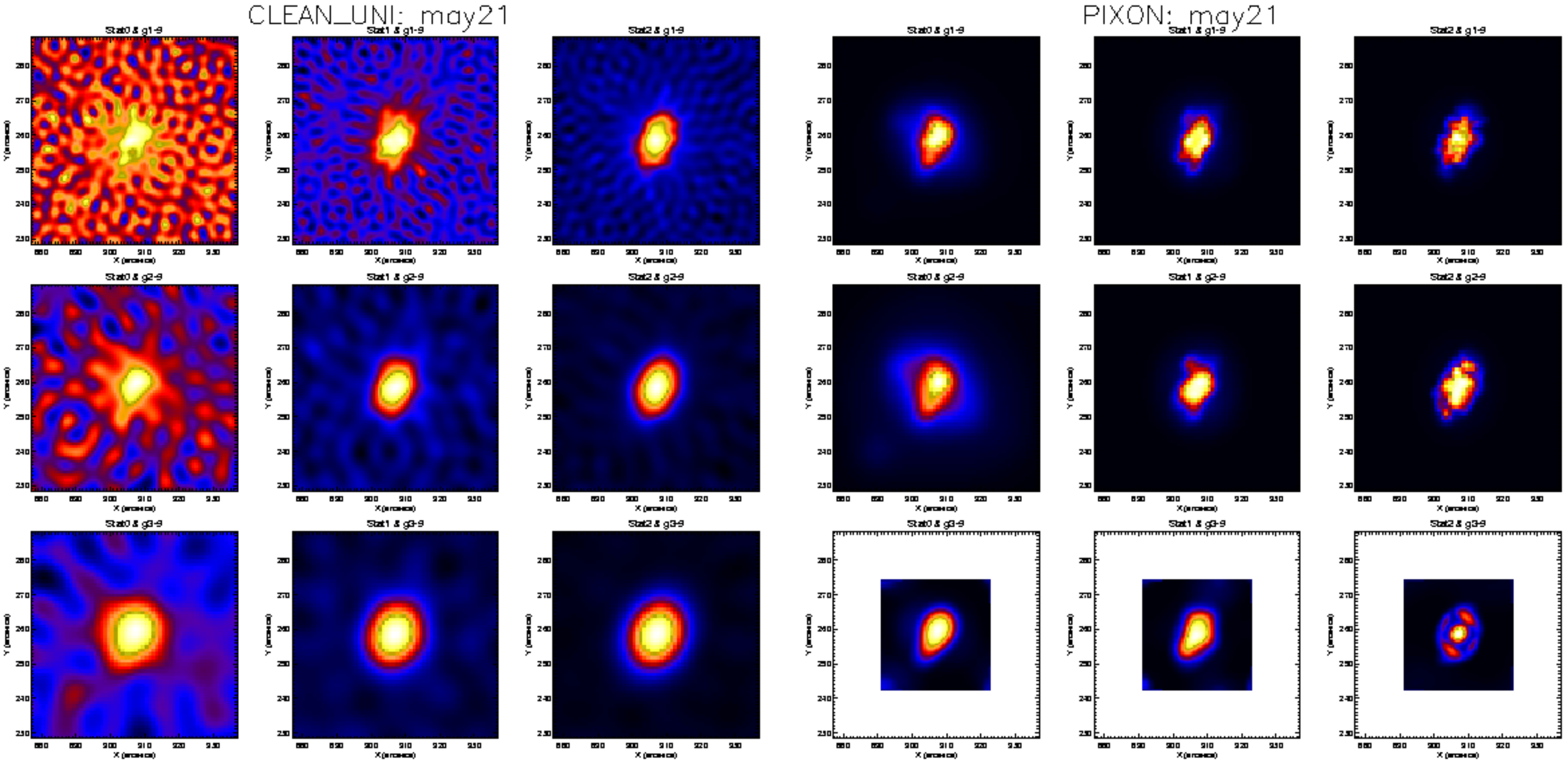


MEM_NJIT

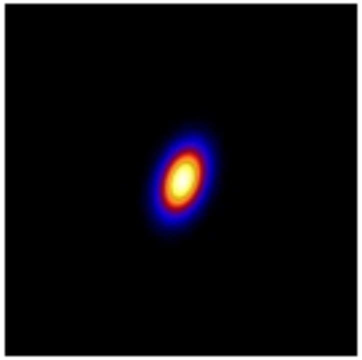


UV_SMOOTH

Sam analysis: Jul 23

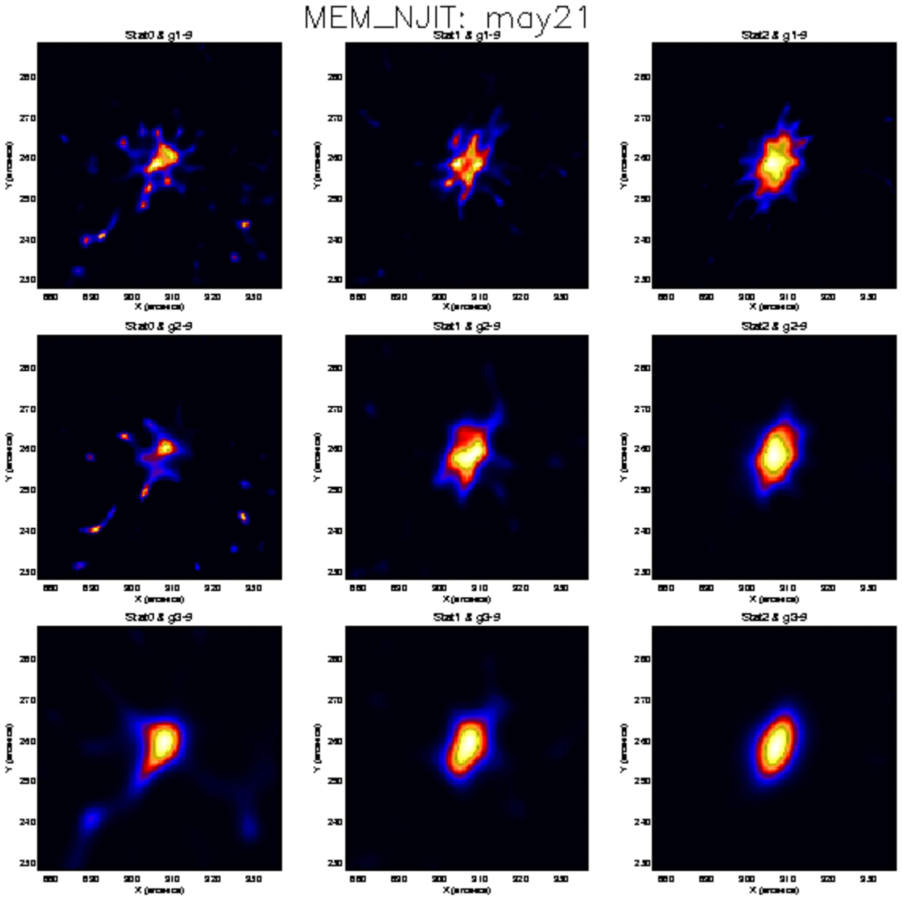


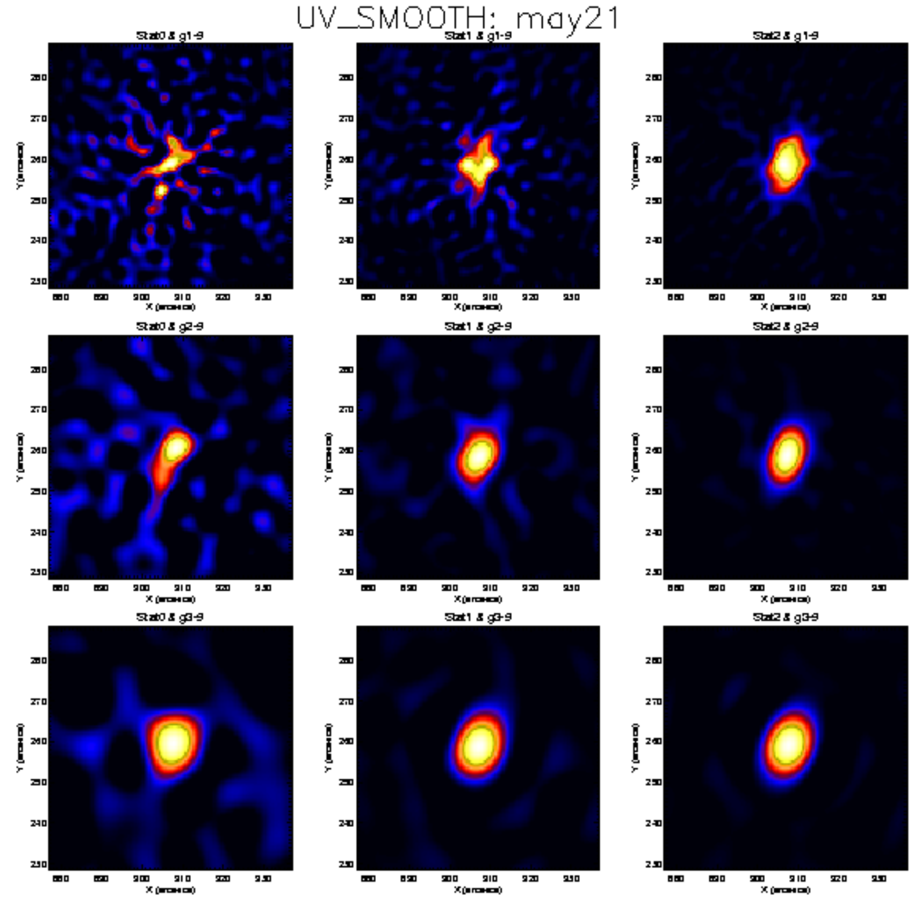
CLEAN



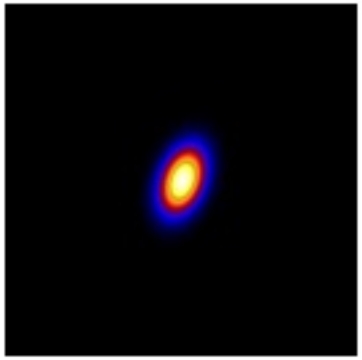
PIXON

Sam analysis: May 21



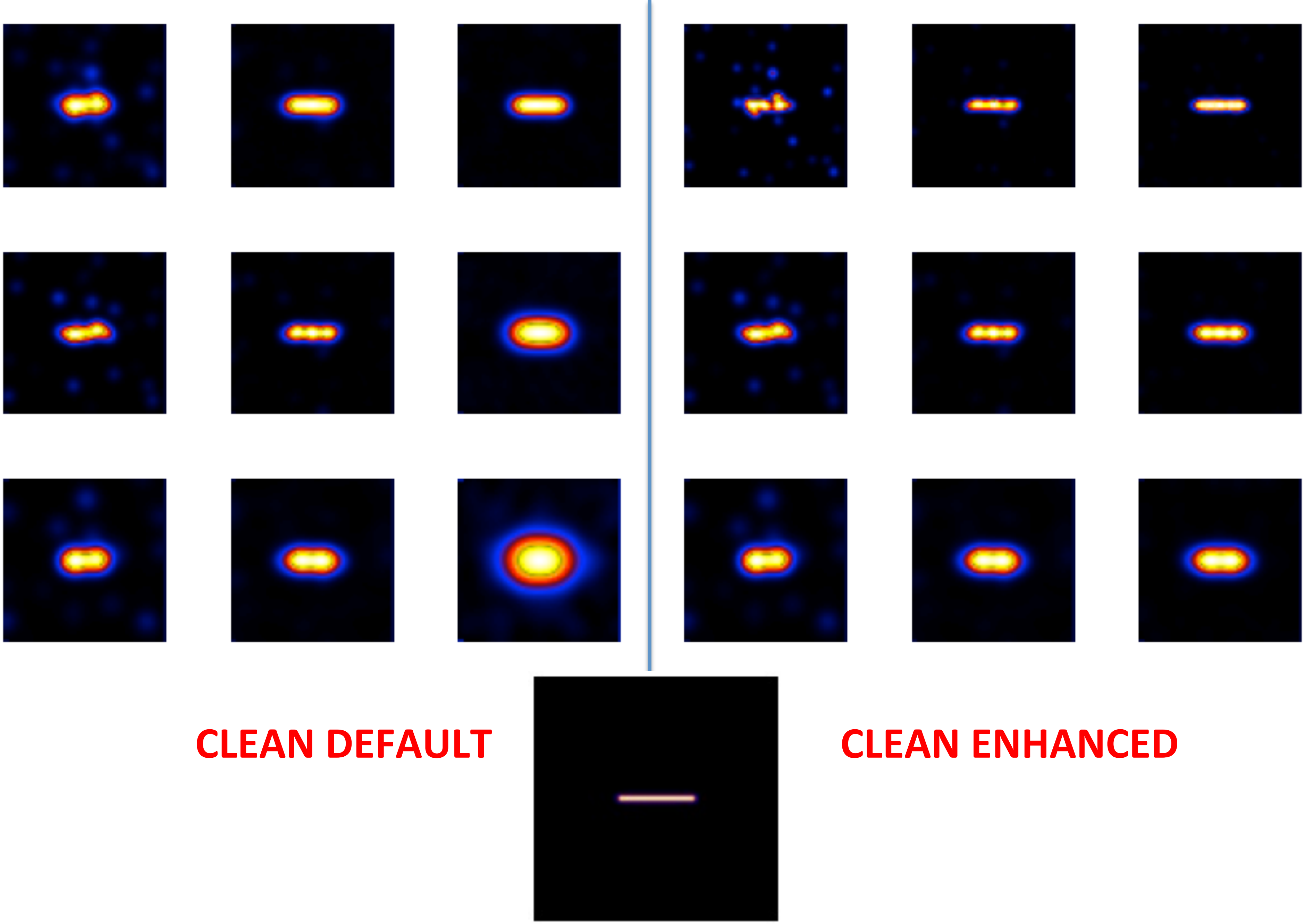


MEM_NJIT

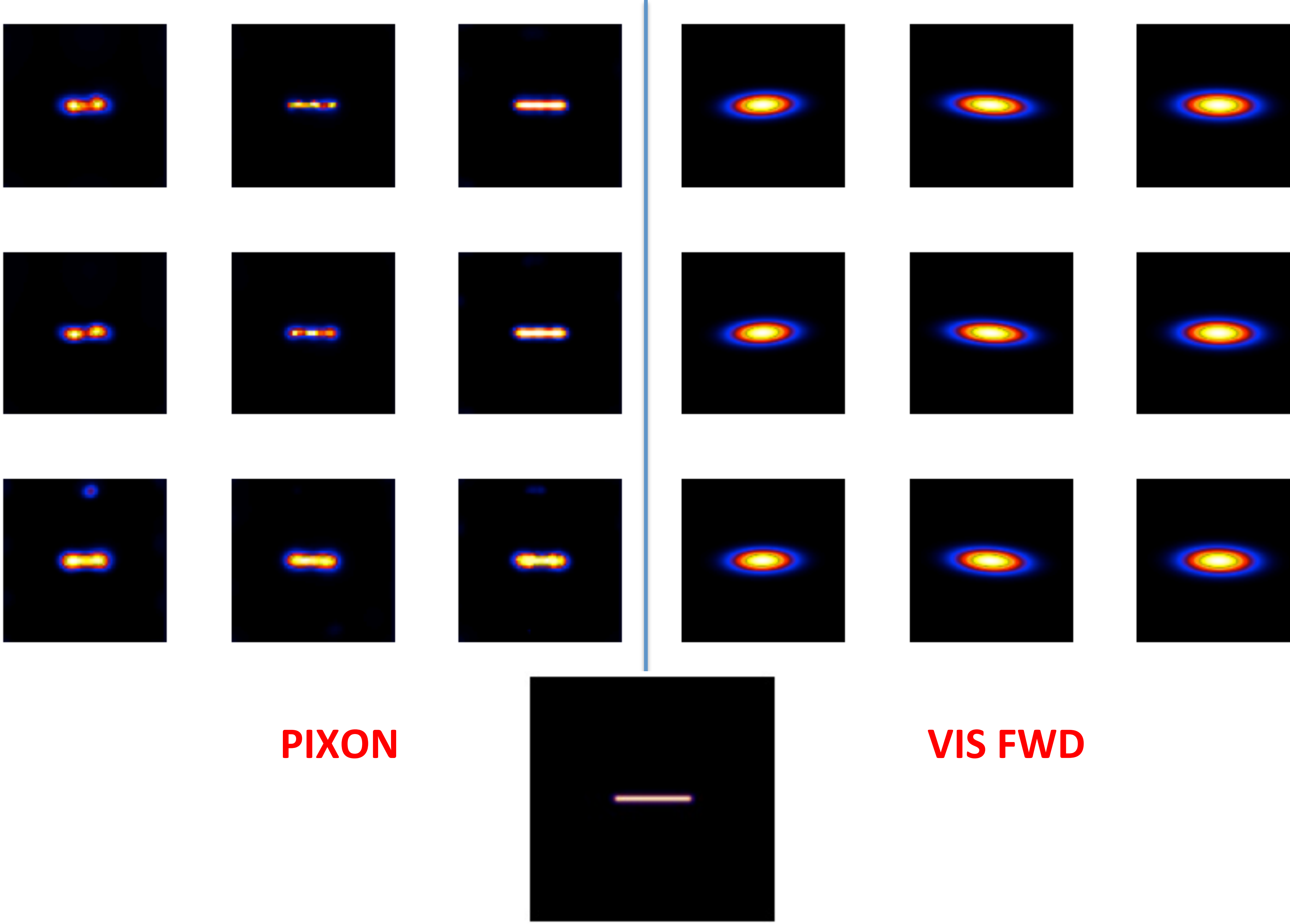


UV_SMOOTH

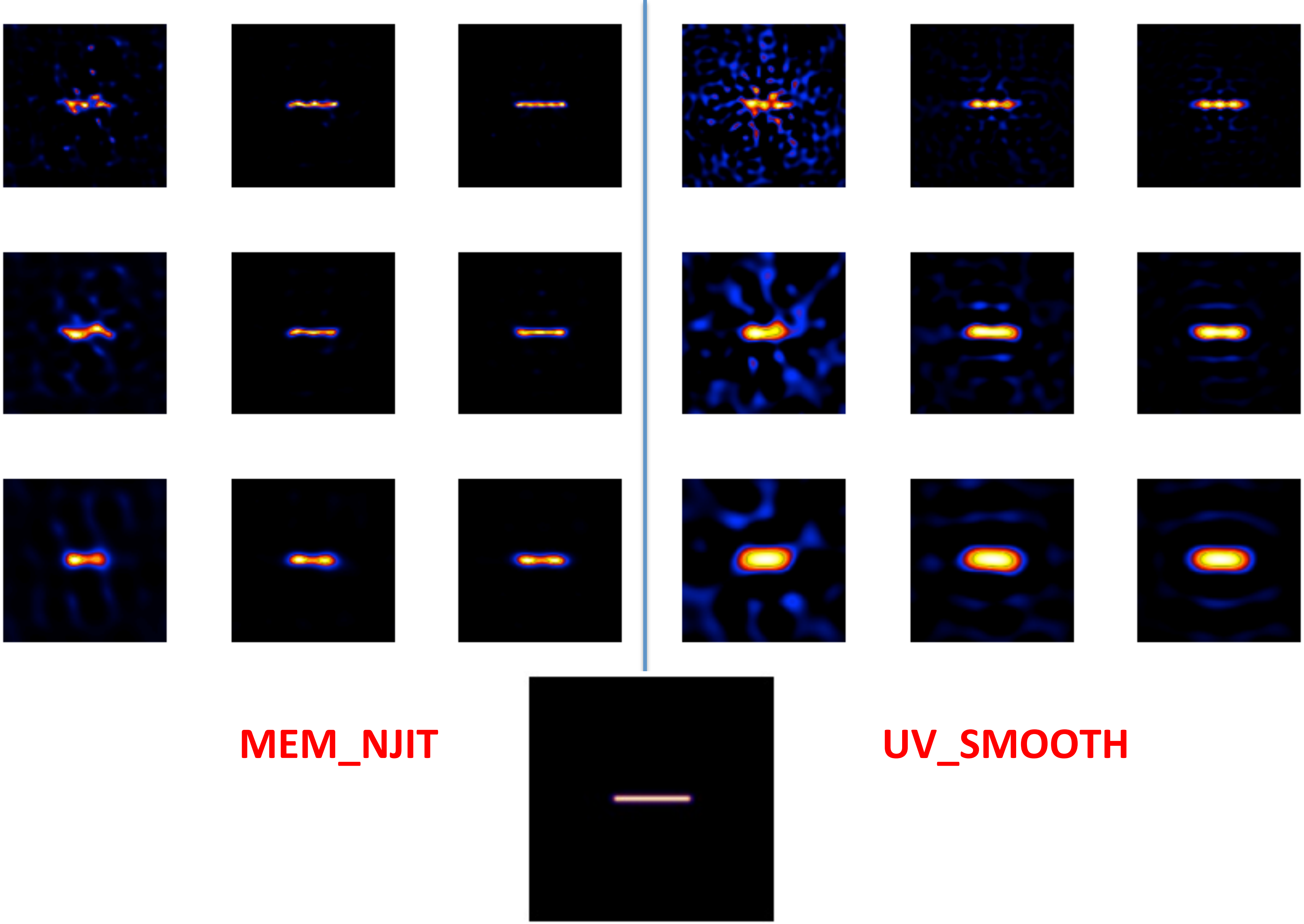
Sam analysis: May 21



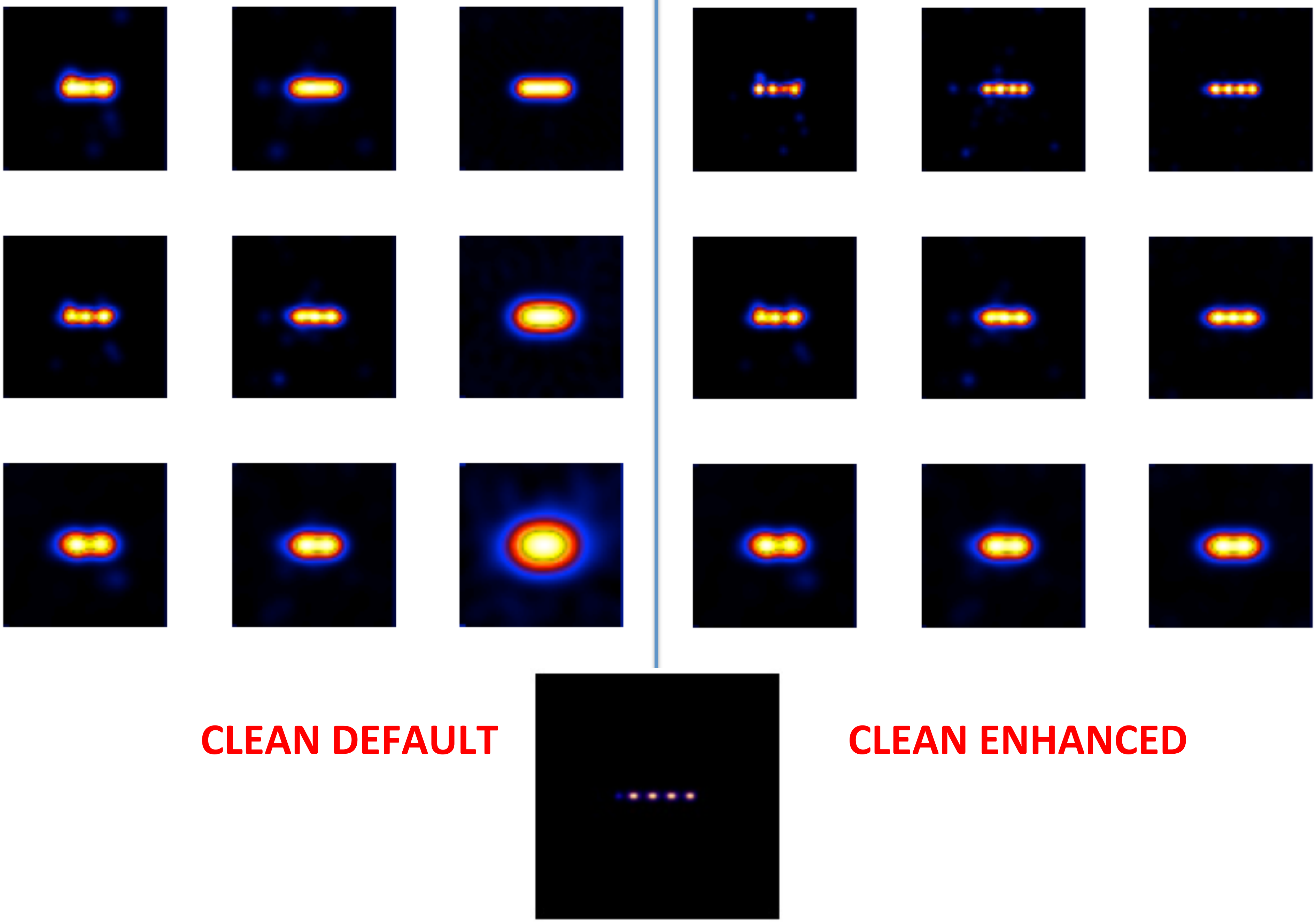
Anna analysis: Line source (constant intensity)



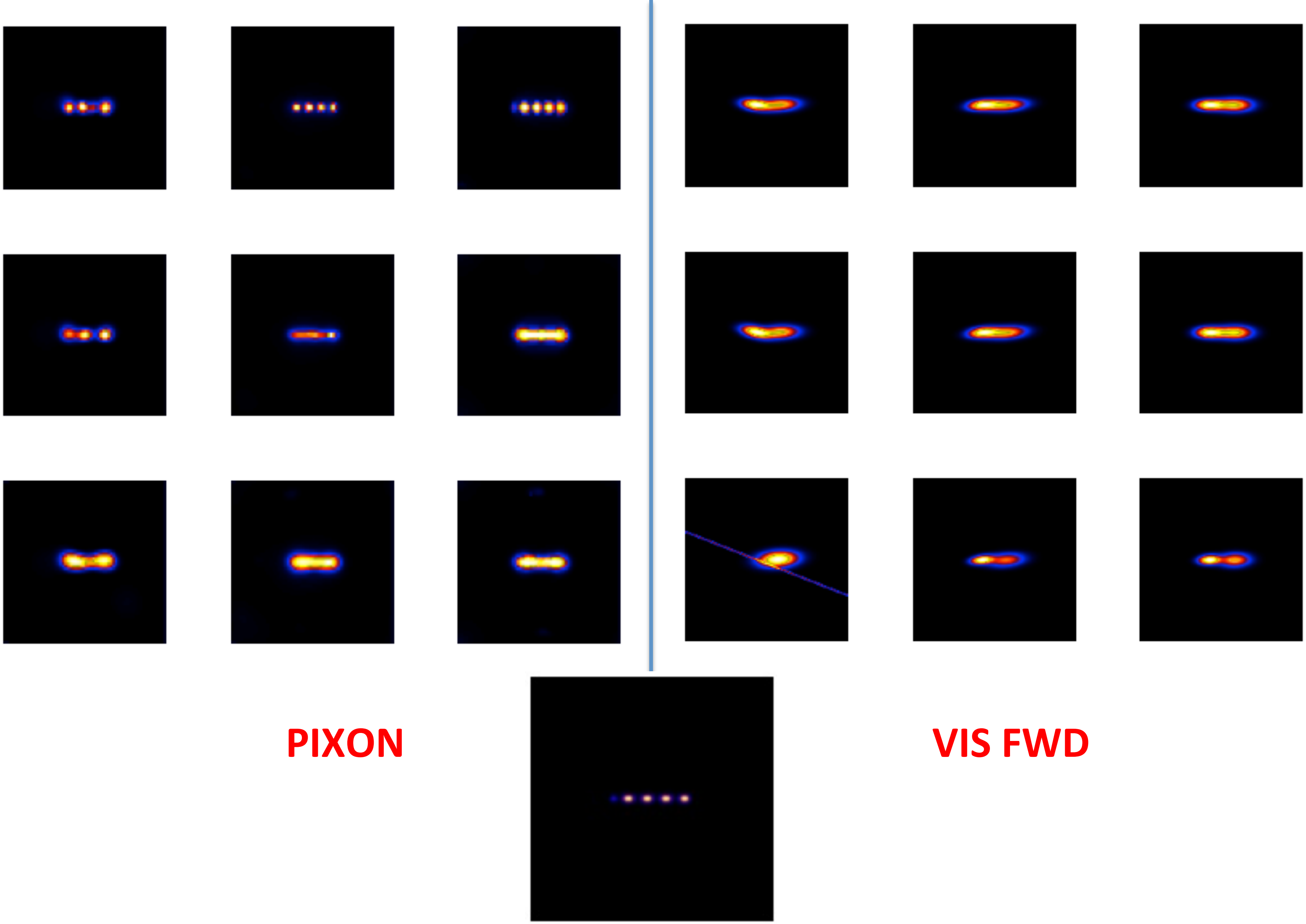
Anna analysis: Line source (constant intensity)



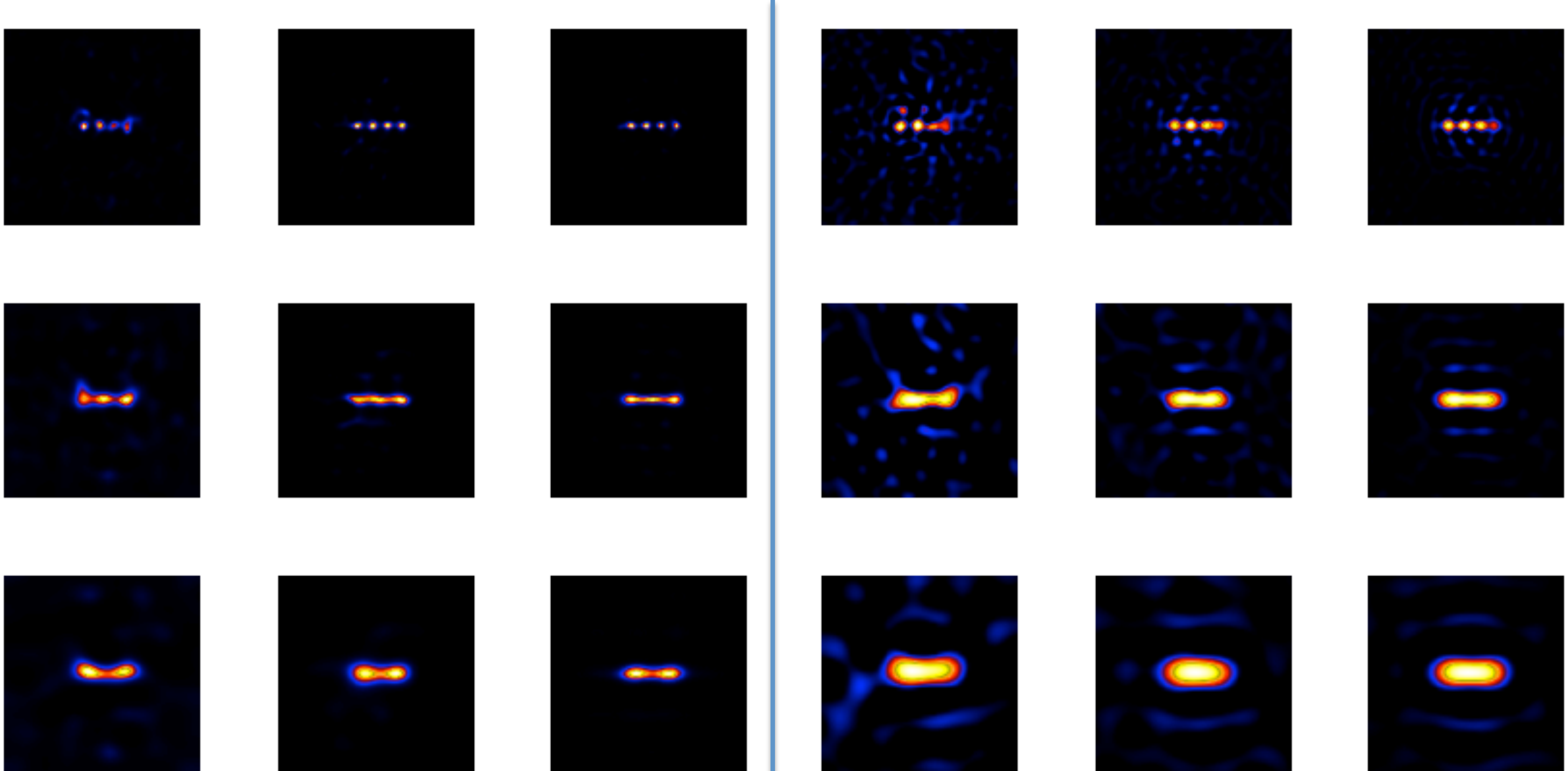
Anna analysis: Line source (constant intensity)



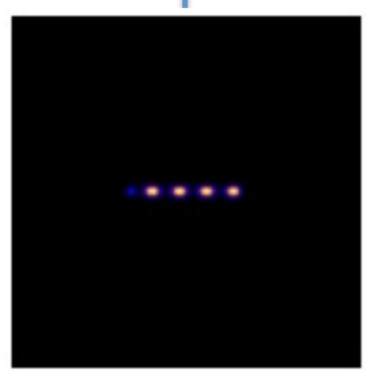
Anna analysis: Line source (varying intensity)



Anna analysis: Line source (varying intensity)

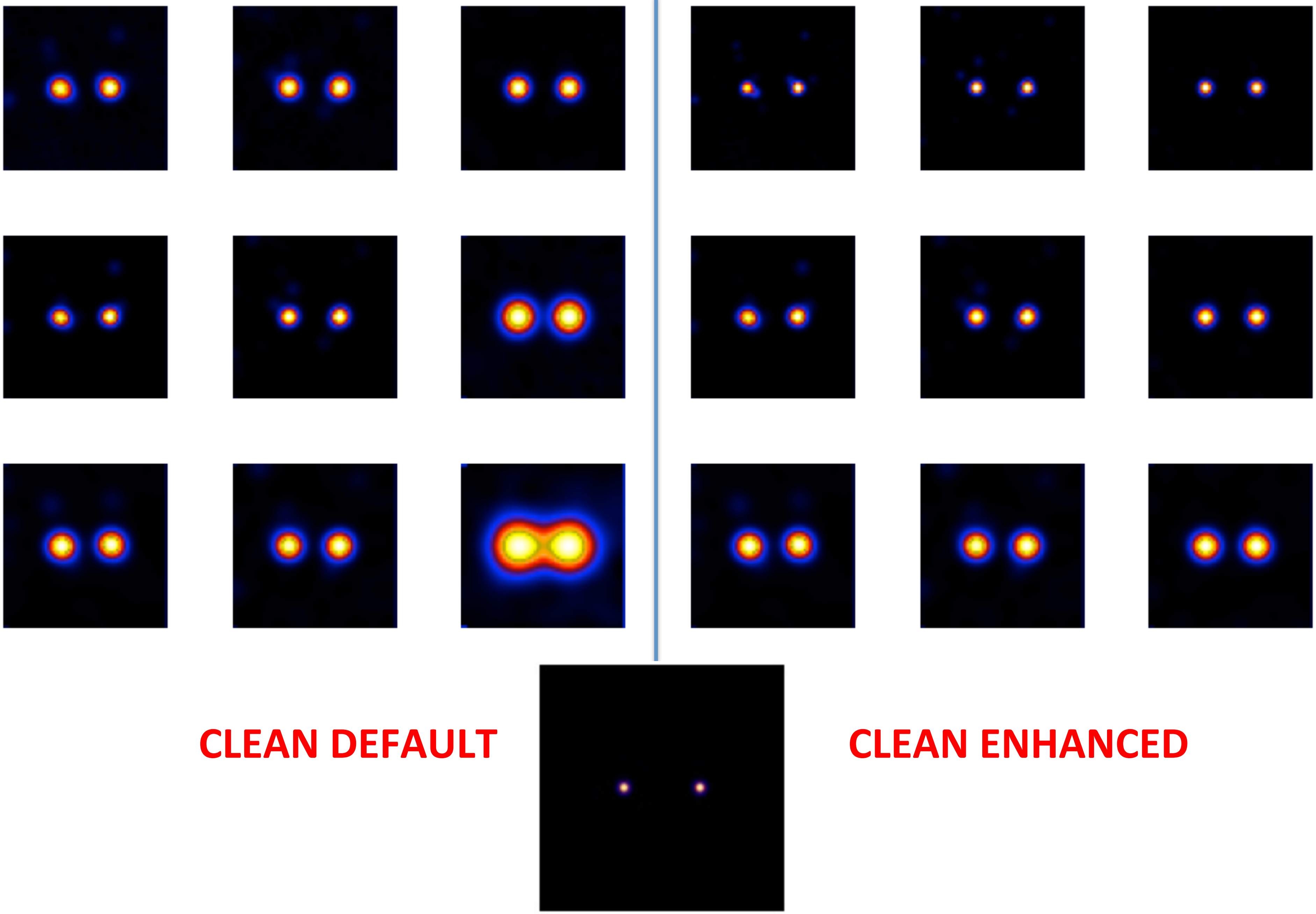


MEM_NJIT

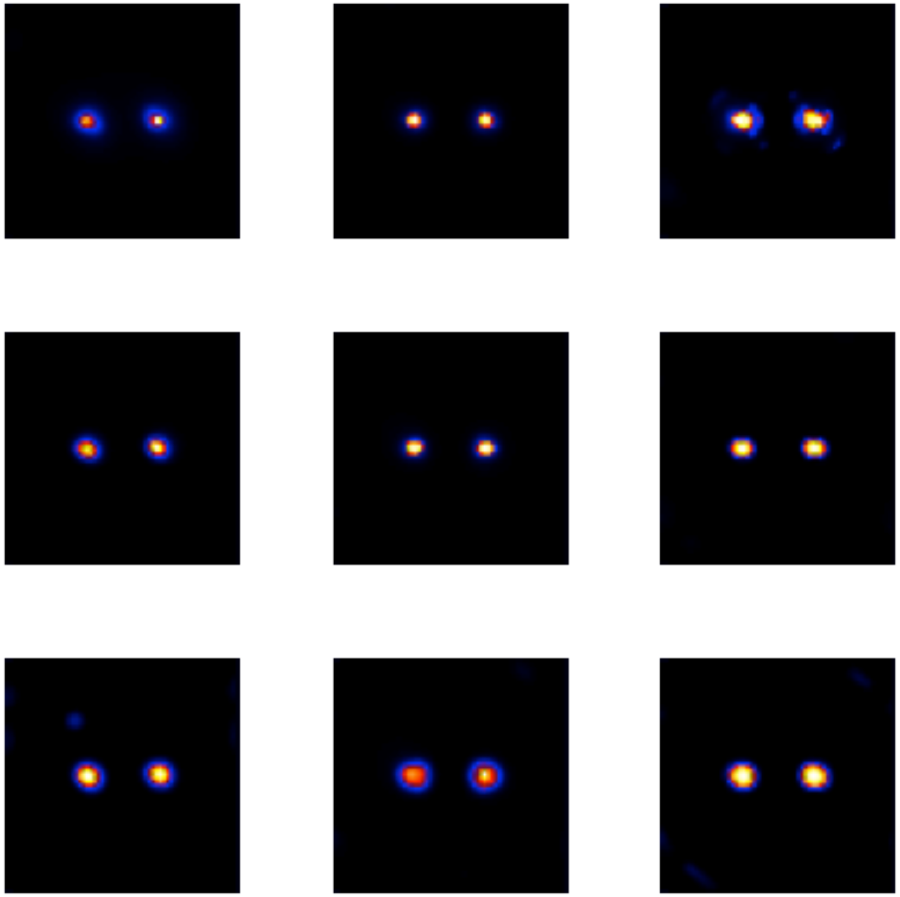


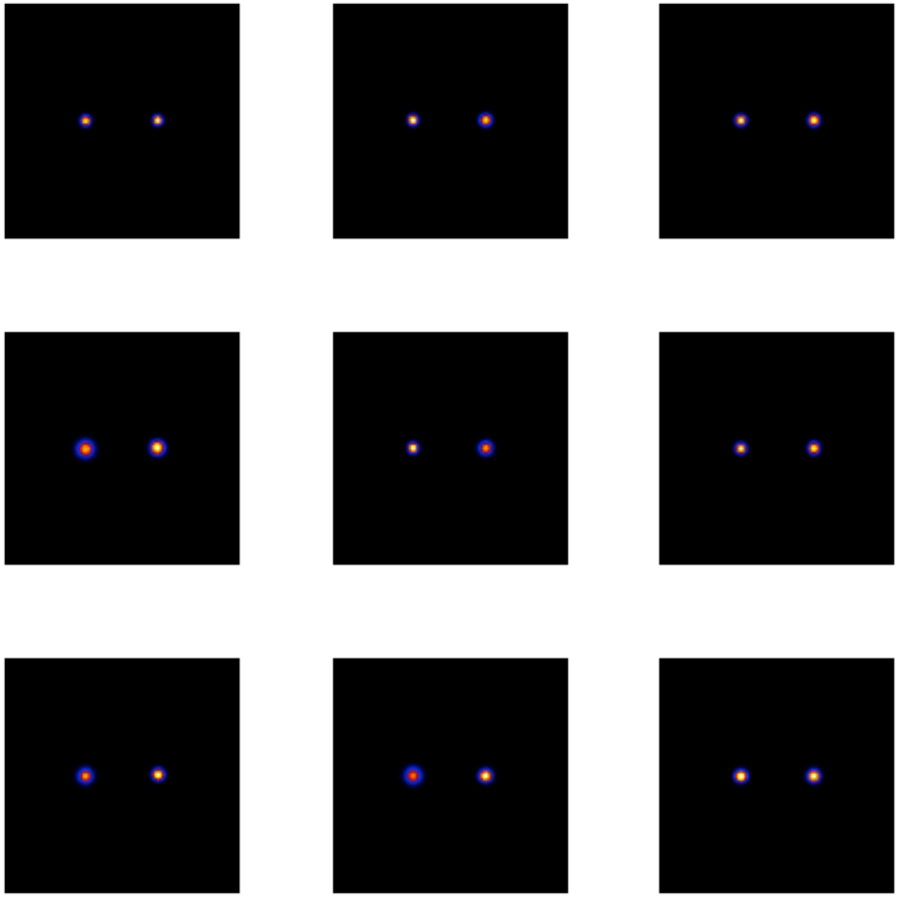
UV_SMOOTH

Anna analysis: Line source (varying intensity)



Anna analysis: Dynamic range (Flux ratio=1)

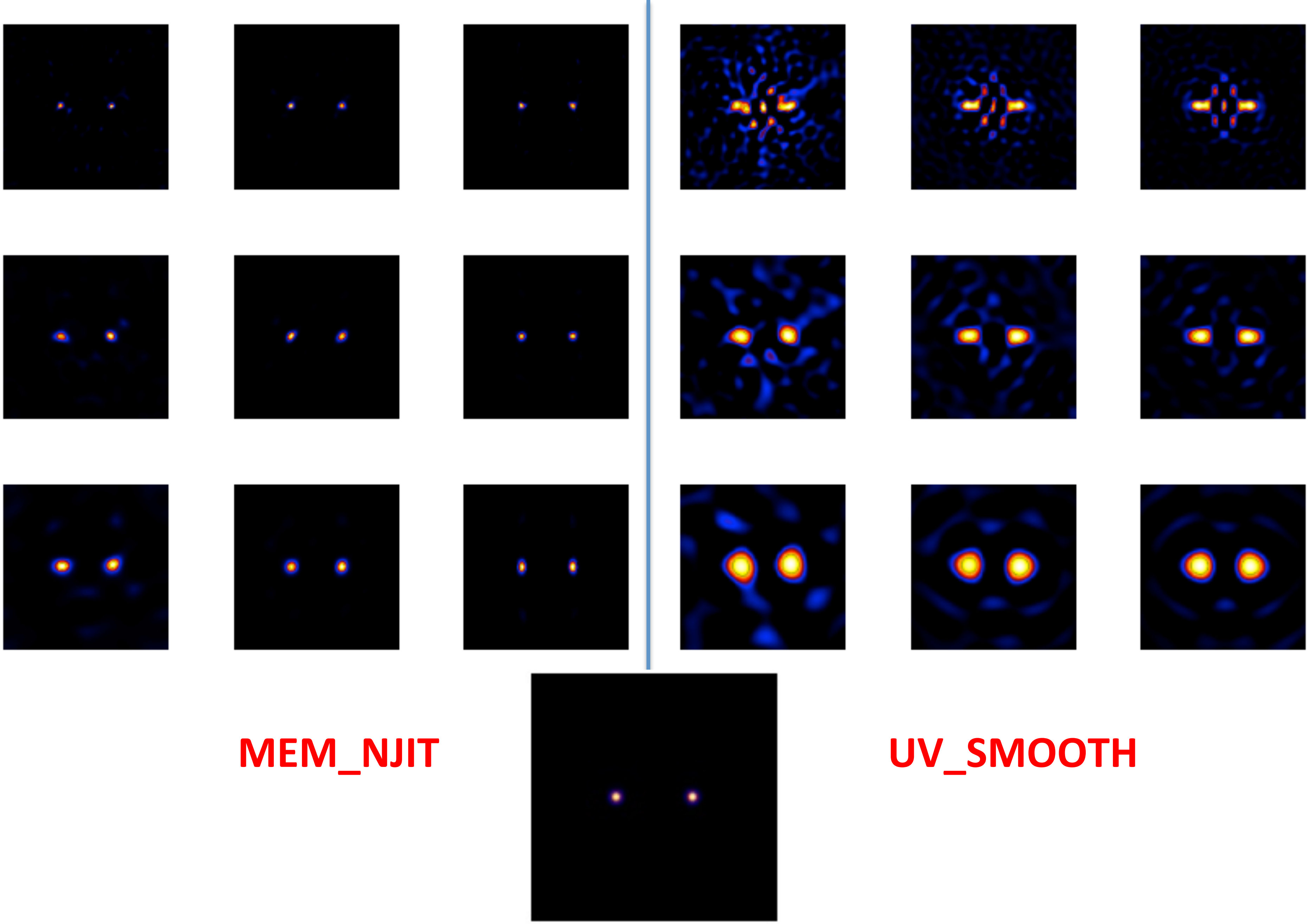




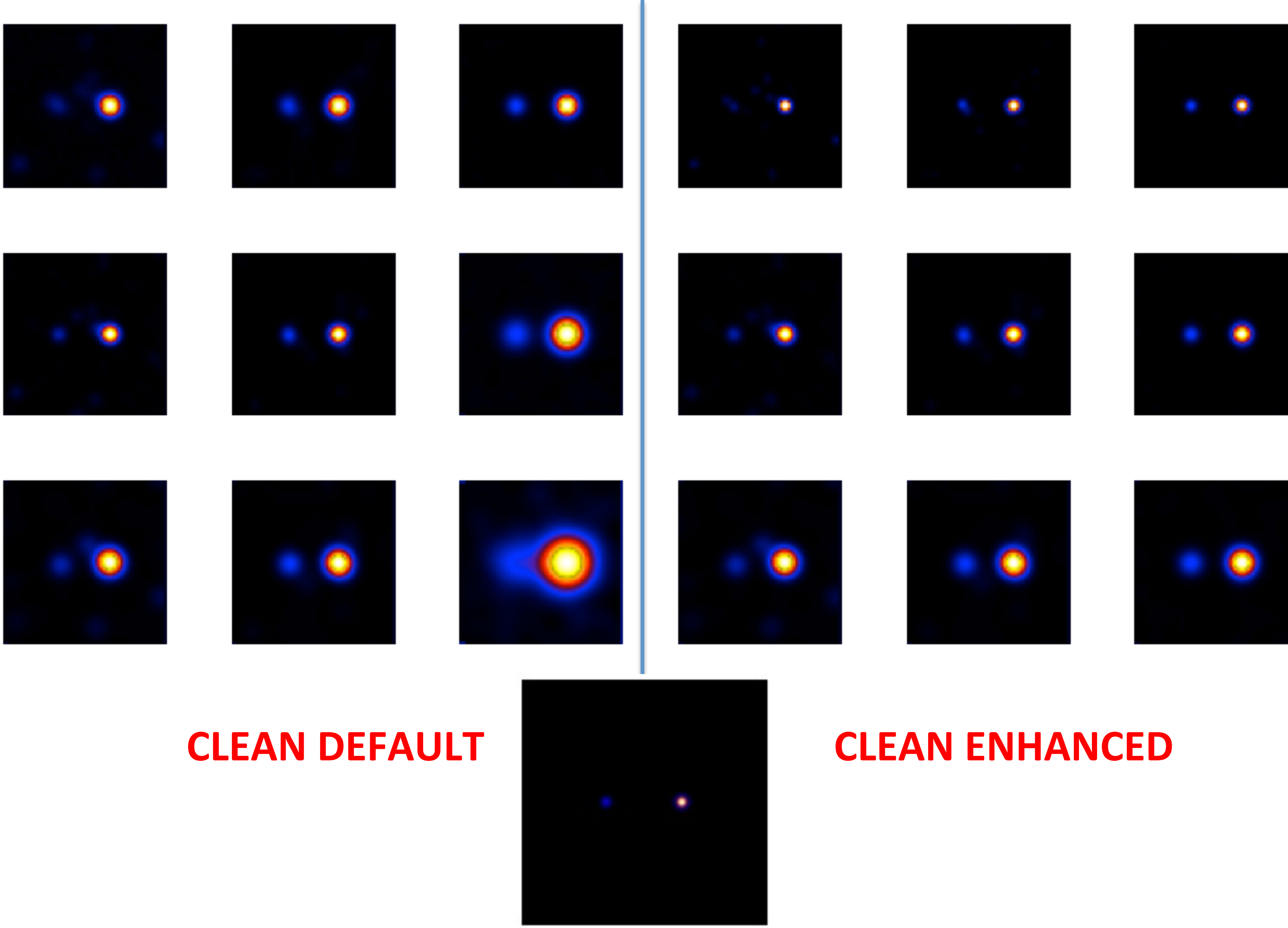
PIXON

VIS FWD

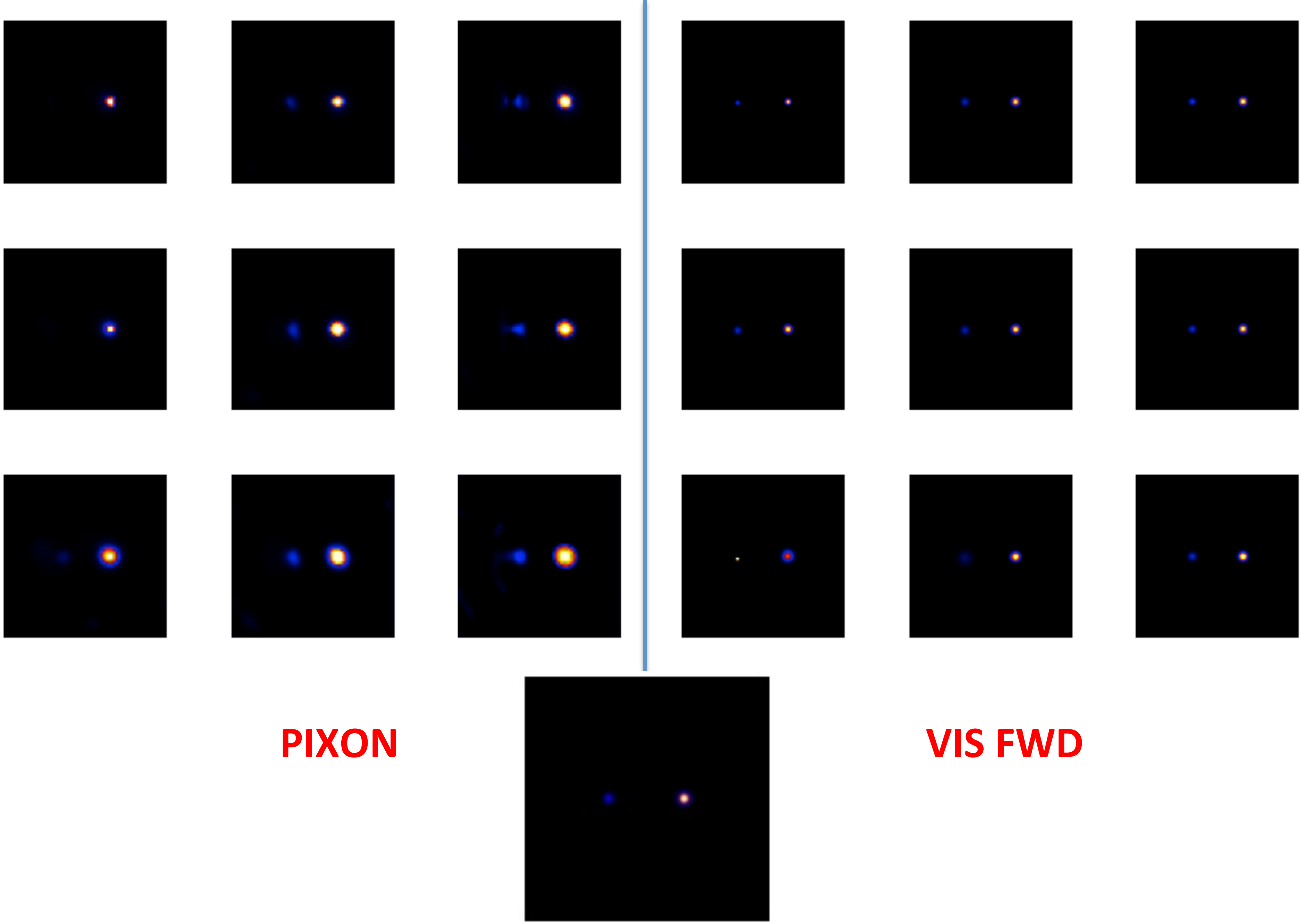
Anna analysis: Dynamic range (Flux ratio = 1)



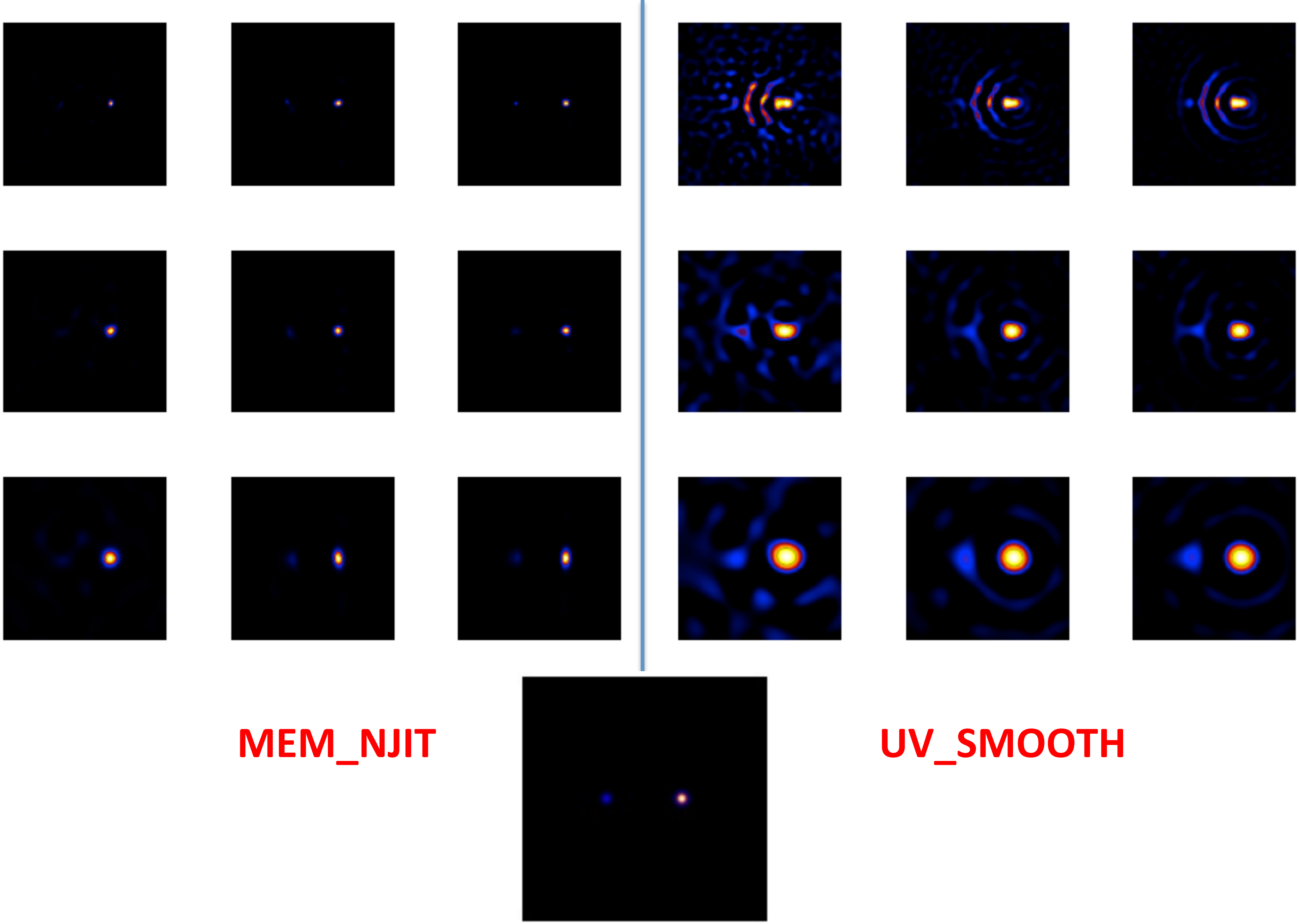
Anna analysis: Dynamic range (Flux ratio = 1)



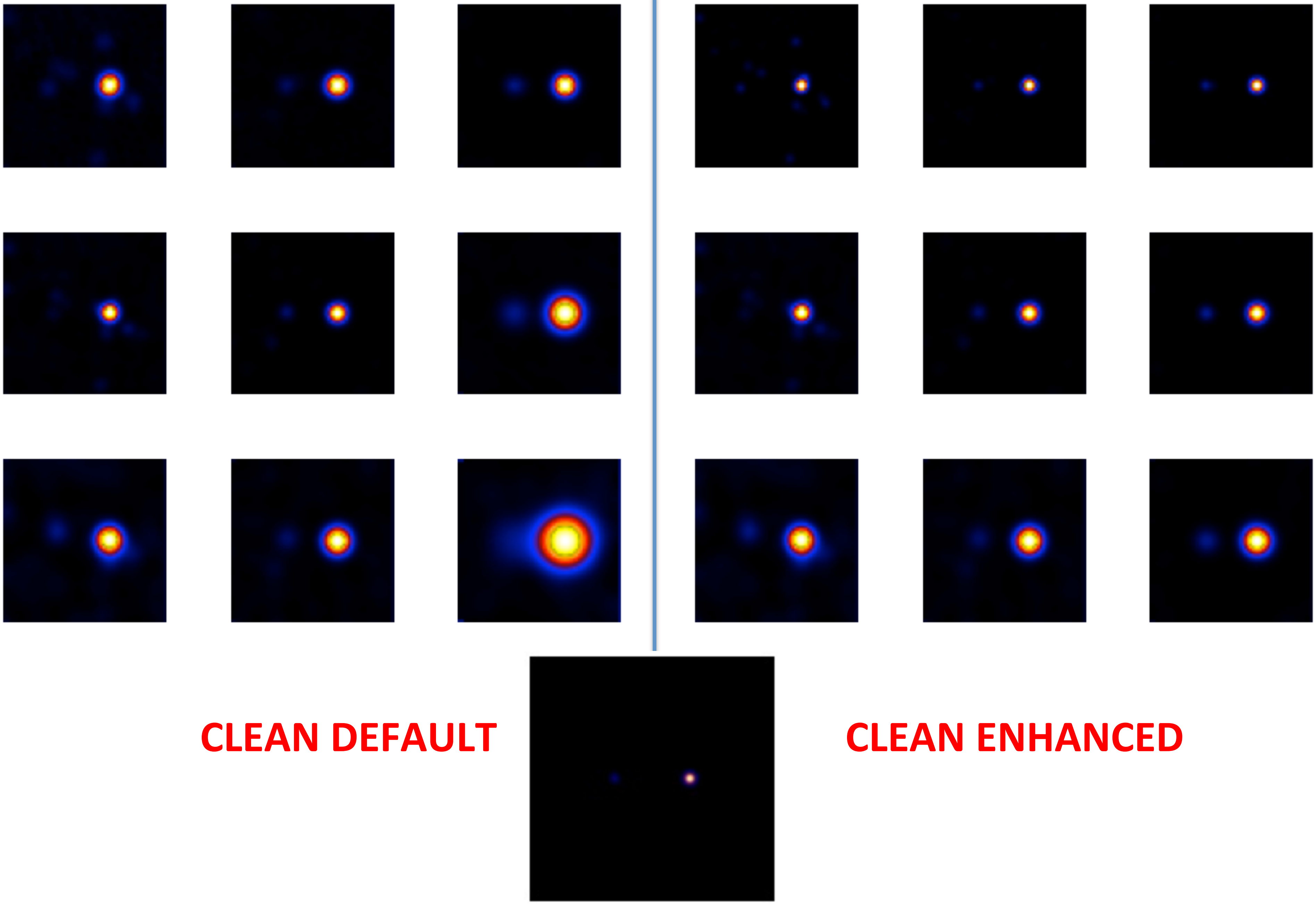
Anna analysis: Dynamic range (Flux ratio = 5)



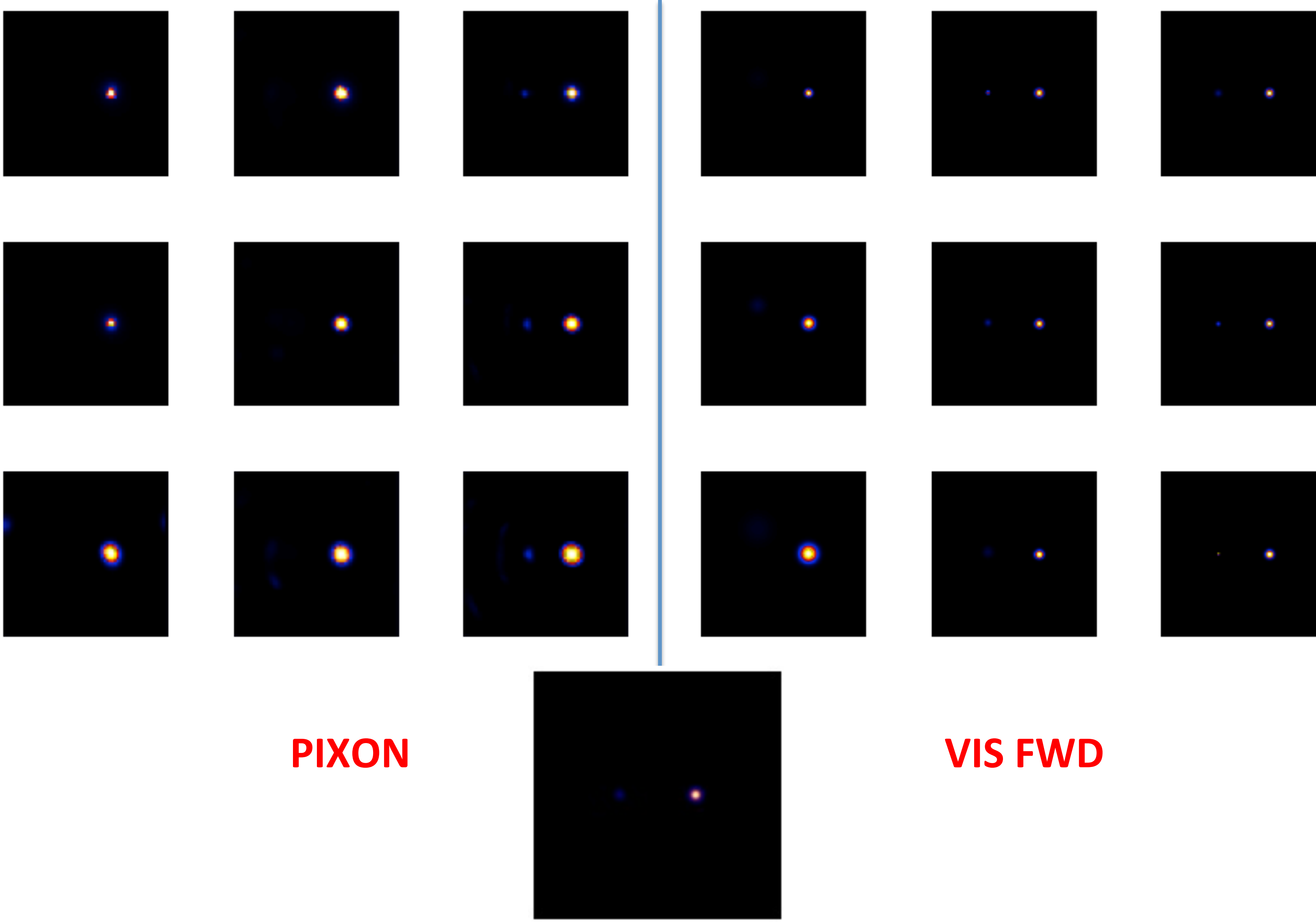
Anna analysis: Dynamic range (Flux ratio = 5)



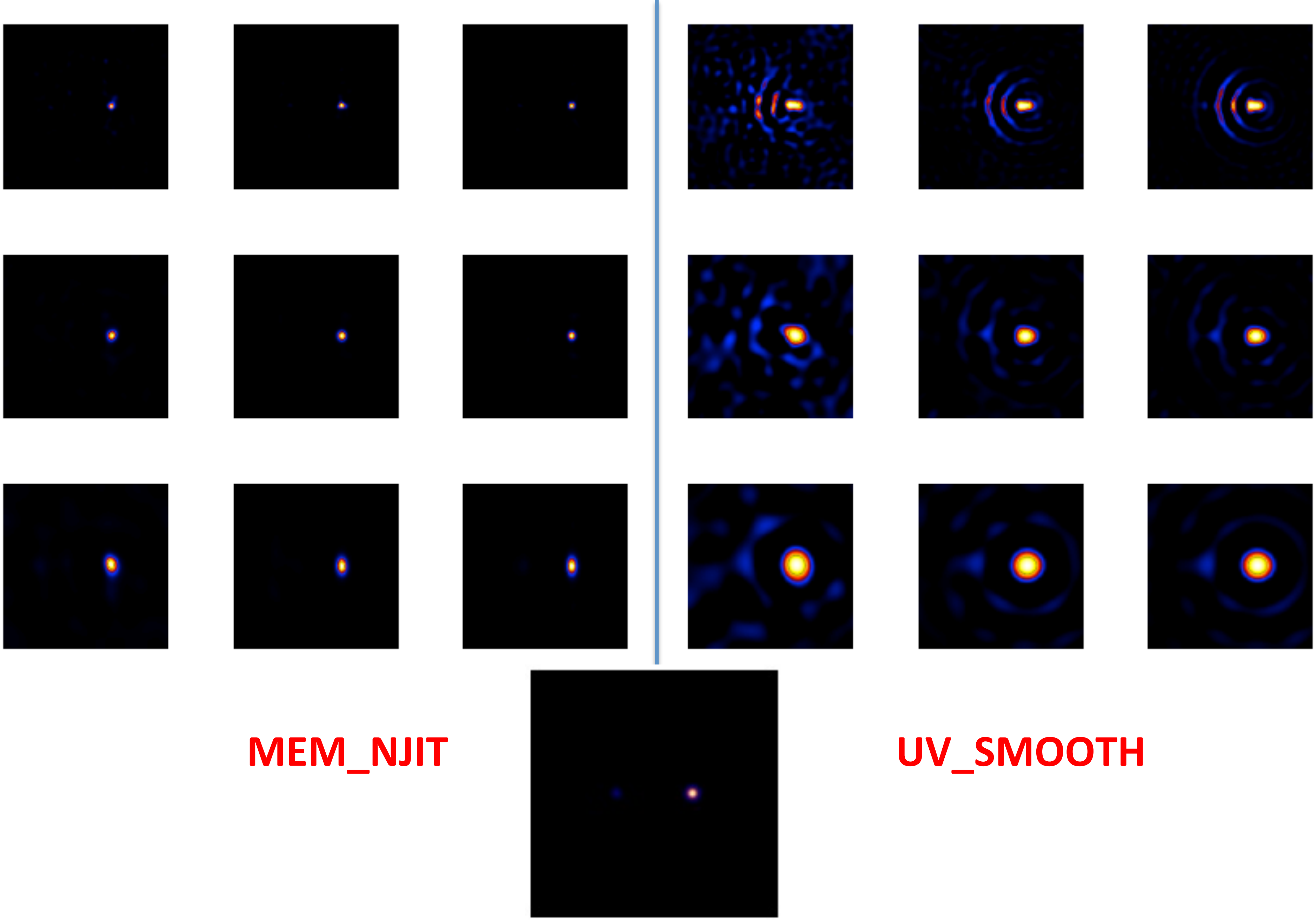
Anna analysis: Dynamic range (Flux ratio = 5)



Anna analysis: Dynamic range (Flux ratio = 10)

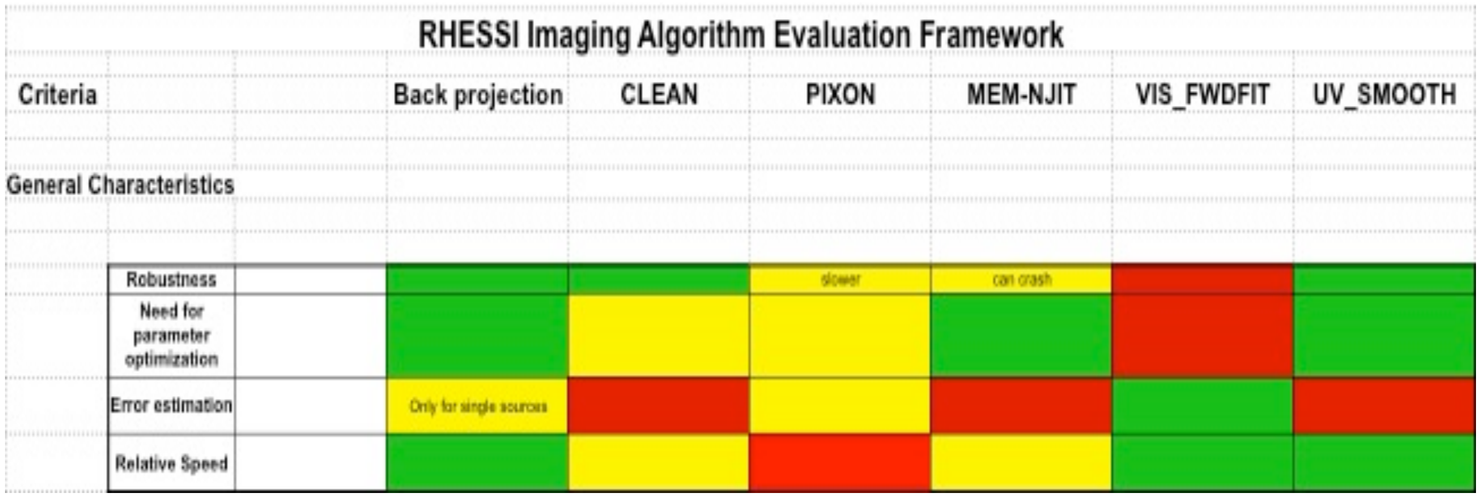


Anna analysis: Dynamic range (Flux ratio = 10)



Anna analysis: Dynamic range (Flux ratio = 10)

Lookup table connecting each algorithm to a specific imaging parameter

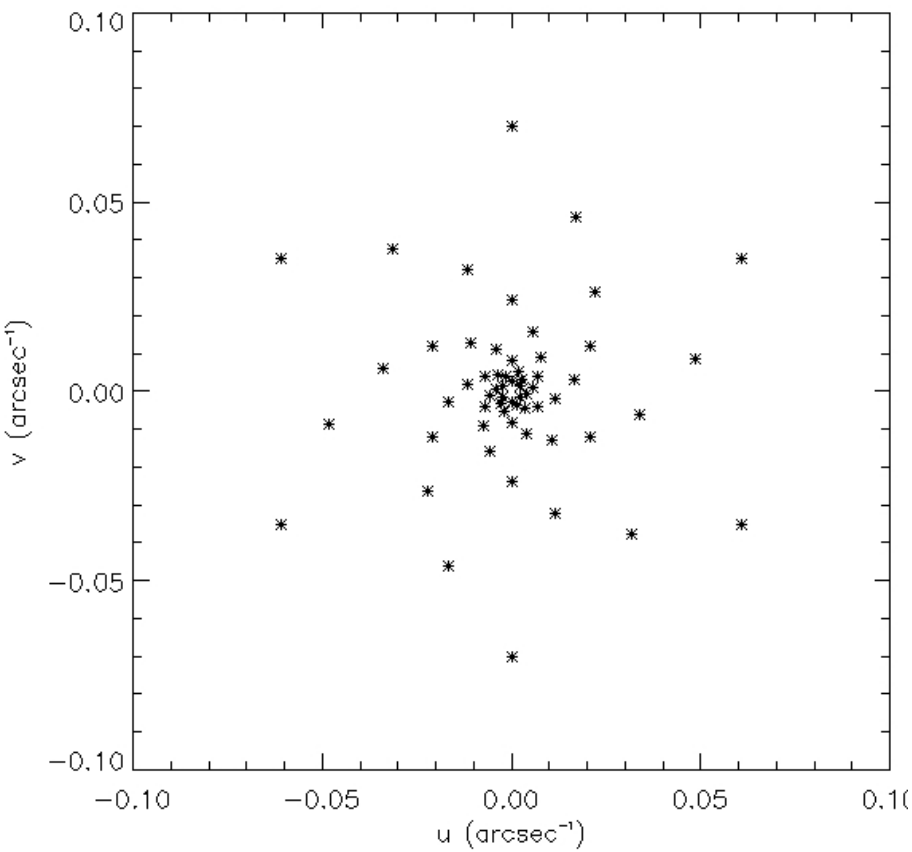


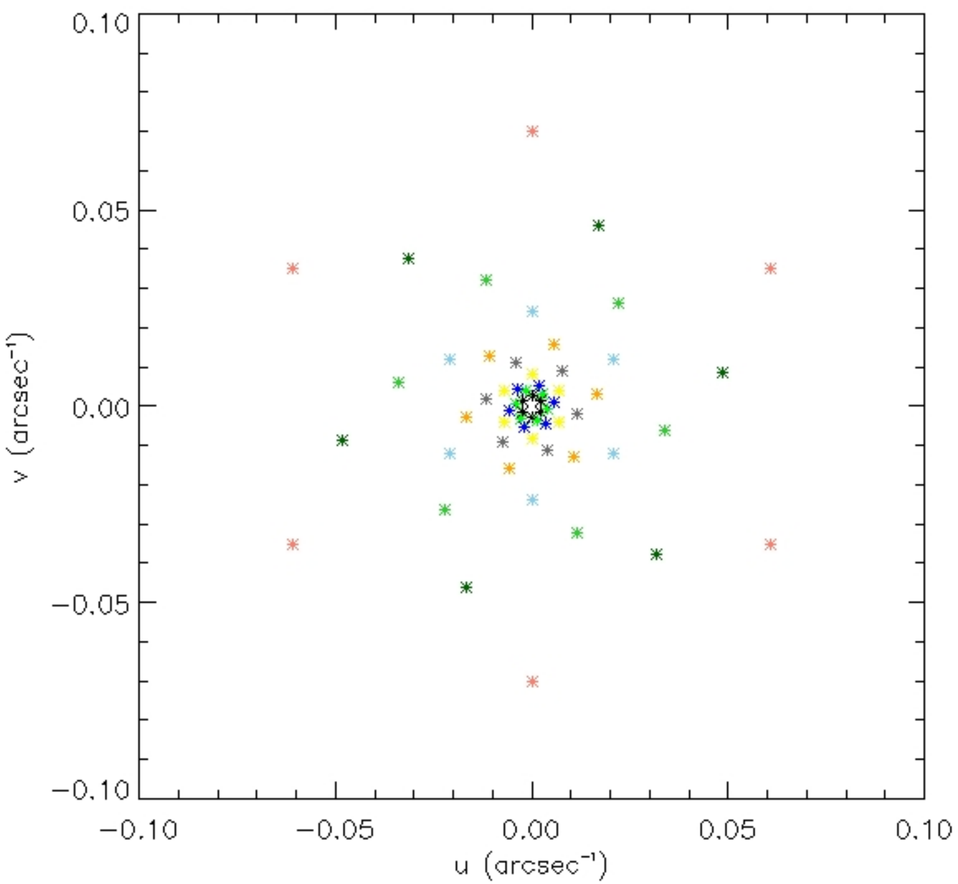
RHESSI Imaging Algorithm Evaluation Framework

Criteria	Back projection	CLEAN	PIXON	MEM-NJIT	VIS_FWDFIT	UV_SMOOTH
General Characteristics						
Robustness			slower	can crash		
Need for parameter optimization						
Error estimation	Only for single sources					
Relative Speed						

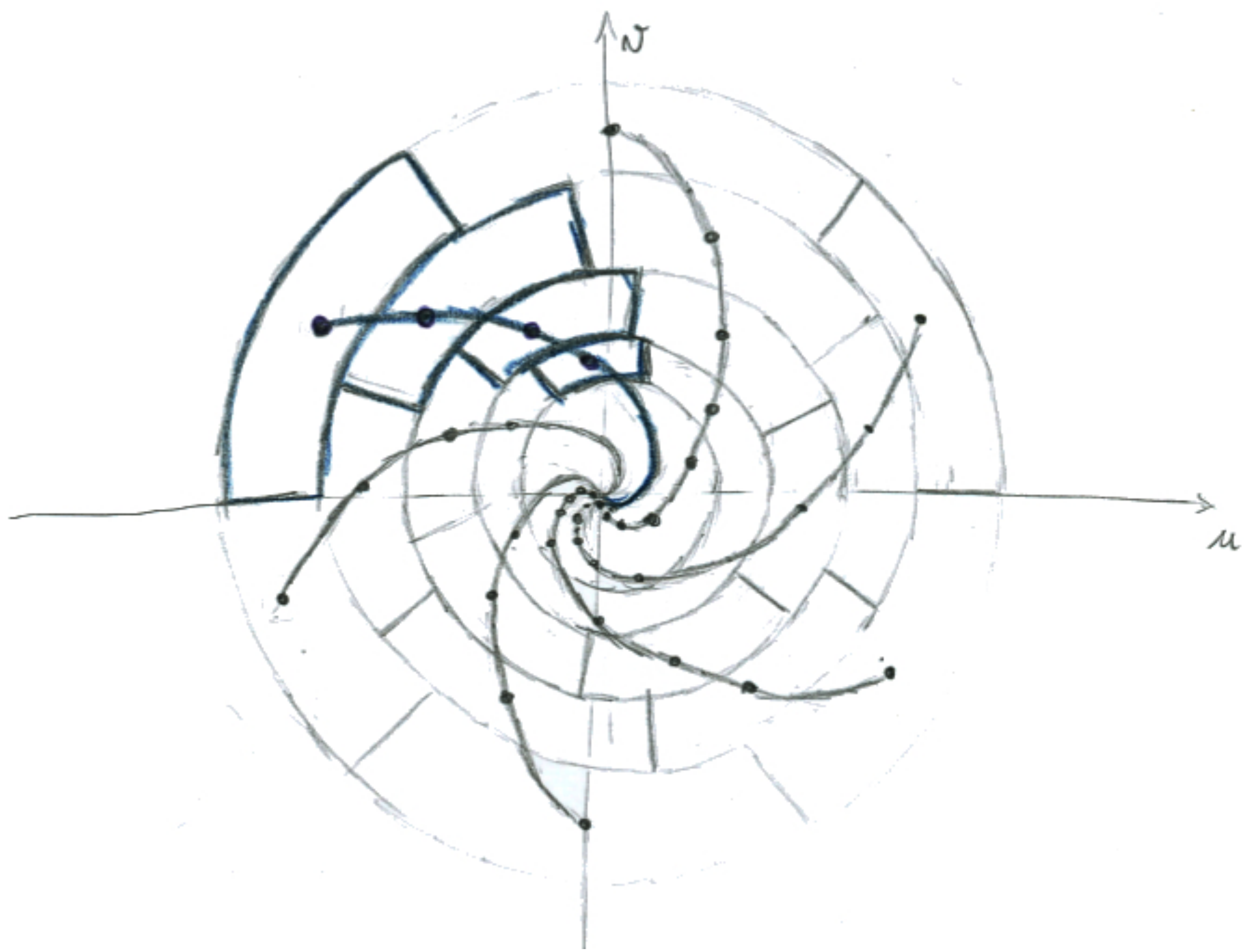
Different colors correspond to different reliability degrees for each imaging property (green: high degree; yellow: handle with care; red: low degree).

Polar Back-Projection



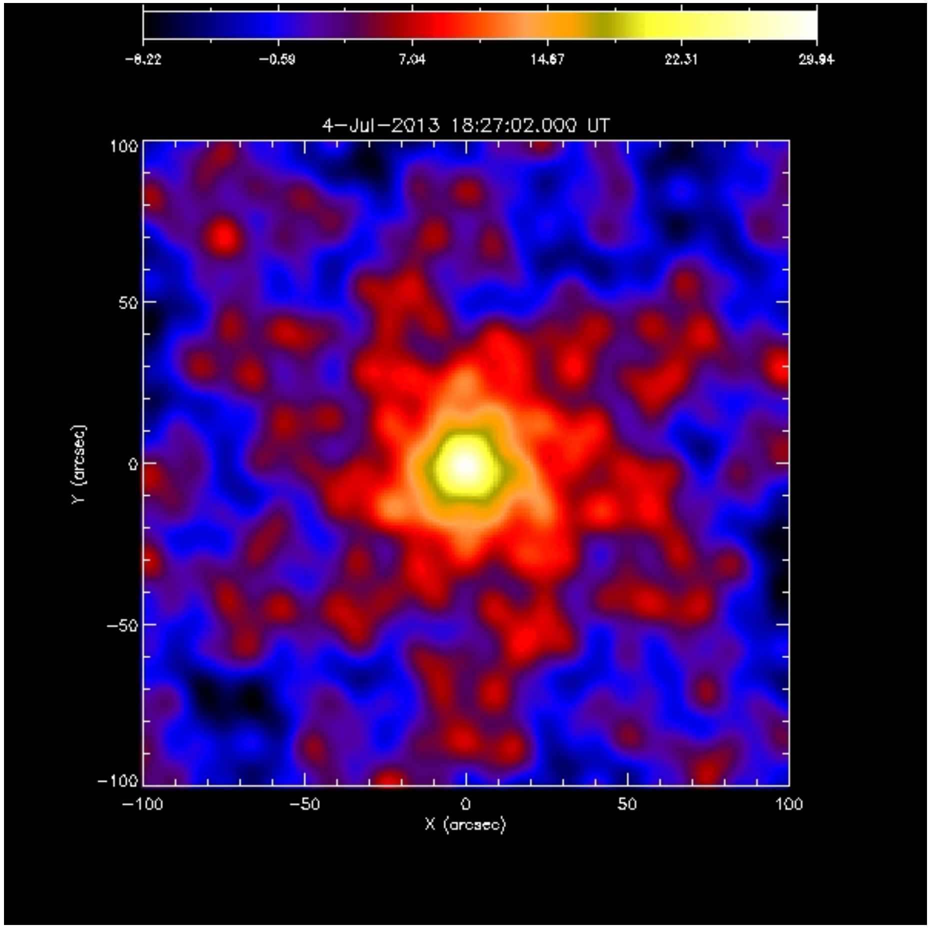


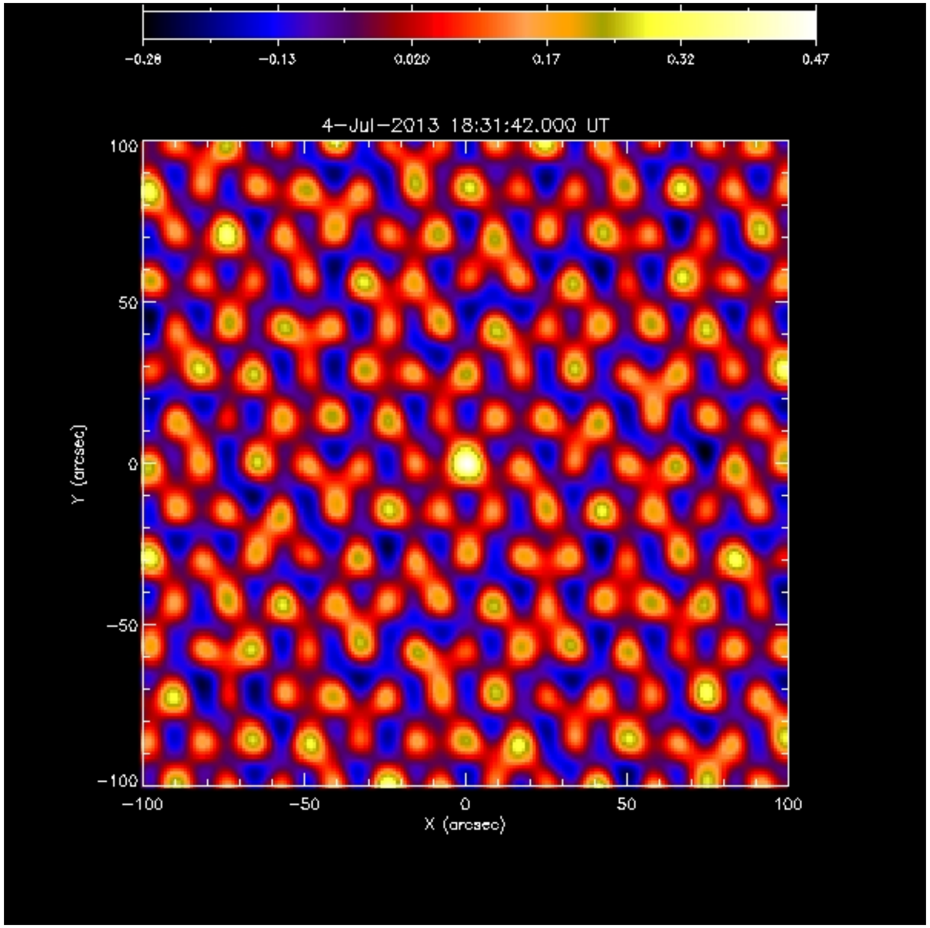
Polar Back-Projection
(Gordon Emslie)



Bpj vs Polar Bpj

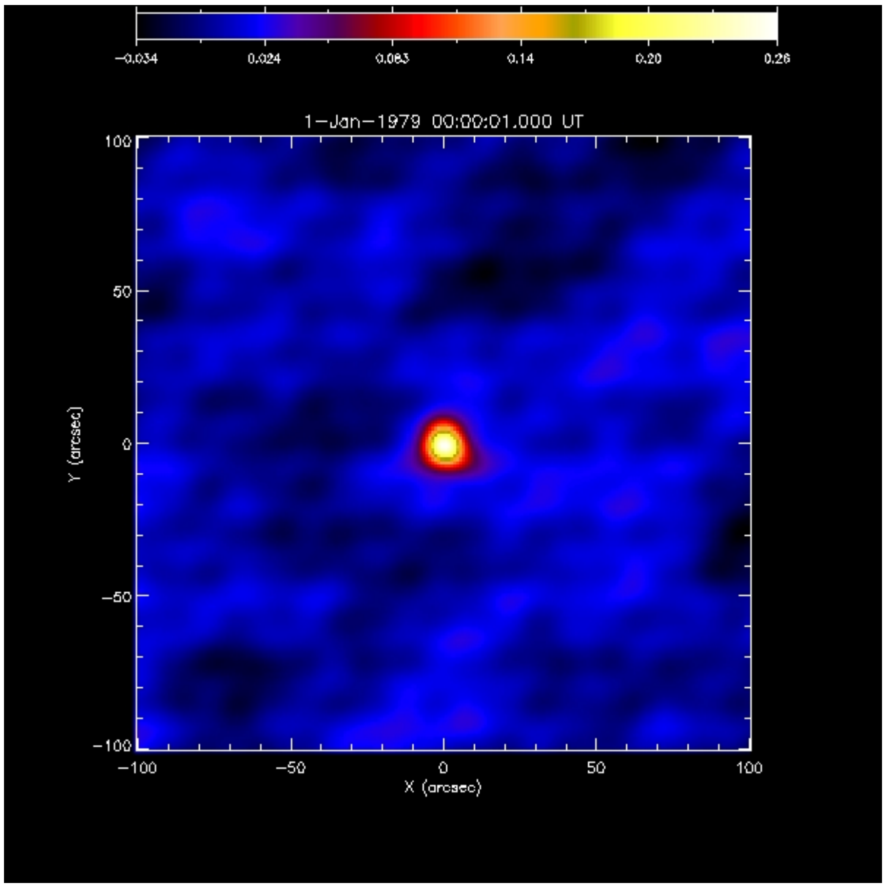
Low statistics

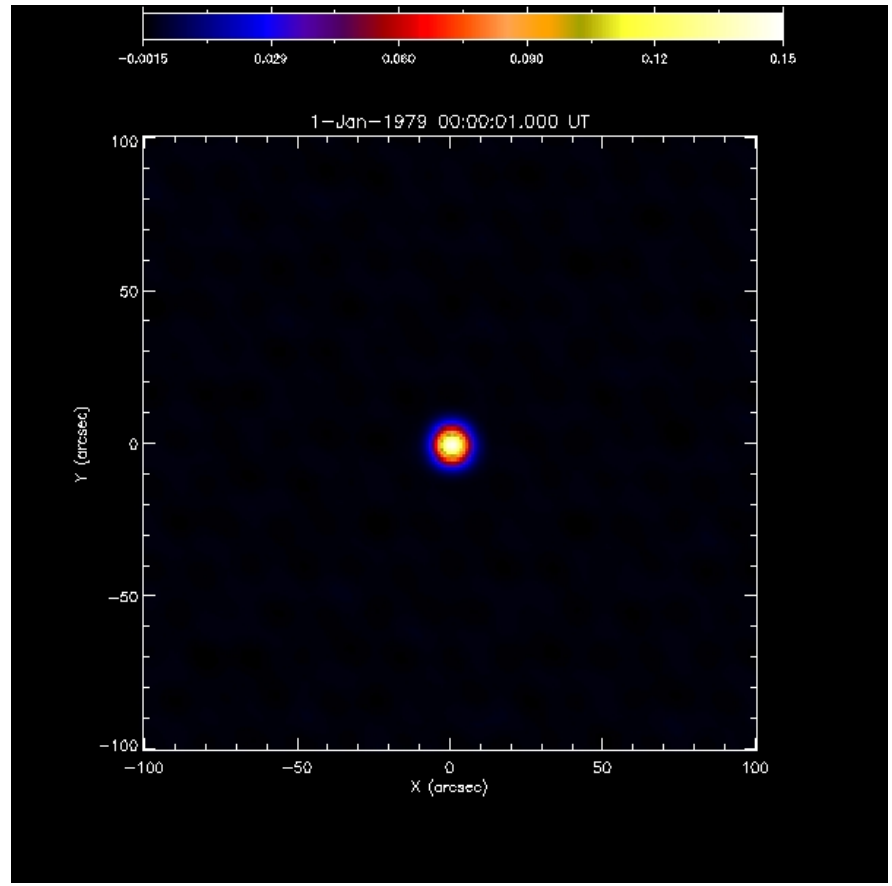




Vis-Clean results

Low statistics





expectation-maximization

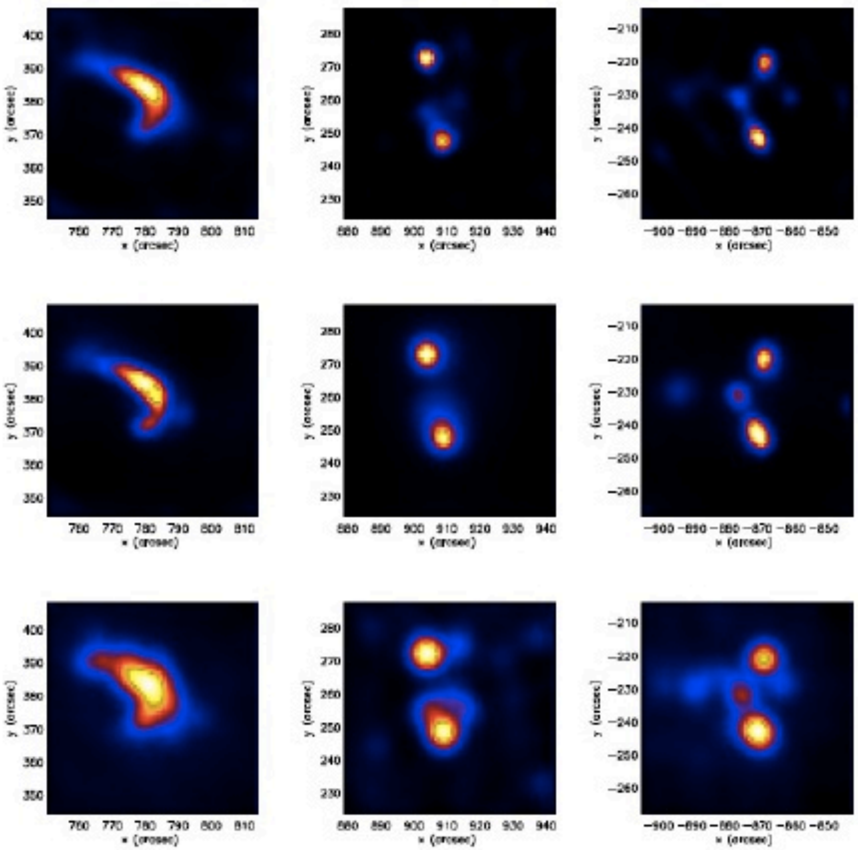
$$f_{k+1} = f_k \frac{A^T\left(\frac{g}{Af_k}\right)}{A^T 1}$$ solves a fixed point problem

either $f_k \to 0$ or $\alpha_k = \frac{A^T\left(\frac{g}{Af_k}\right)}{A^T 1} \to 1$

therefore $z_k := \| f_k A^T \left(1 - \frac{g}{Af_k}\right) \|^2 \to 0$

therefore $z_k = E(z_k)$ is a stopping rule

benvenuto, schwartz, piana and massone, AA, 2013



April 15 2002

	3	4	5	6	7	8
EM	1.360	1.384	1.207	2.207	2.312	5.302
Pixon	1.459	1.507	1.486	2.302	2.464	5.111
CLEAN	14.50	13.98	13.86	27.65	41.07	77.79

February 20 2002

	3	4	5	6	7	8
EM	1.093	1.104	1.037	1.157	1.056	0.882
Pixon	1.212	1.250	1.154	1.167	1.302	1.106
CLEAN	1.725	1.695	1.721	2.194	2.505	3.249

July 23 2002

	3	4	5	6	7	8
EM	1.080	0.973	1.220	1.341	1.690	1.837
Pixon	1.176	1.007	1.224	1.340	1.702	1.797
CLEAN	5.302	5.326	4.711	5.756	3.908	9.878

SMC

$$p(I \mid V_{obs}) = \frac{p_{pr}(I)\, p(V_{obs} \mid I)}{p(V_{obs})}$$

V_{obs} observed visibilities

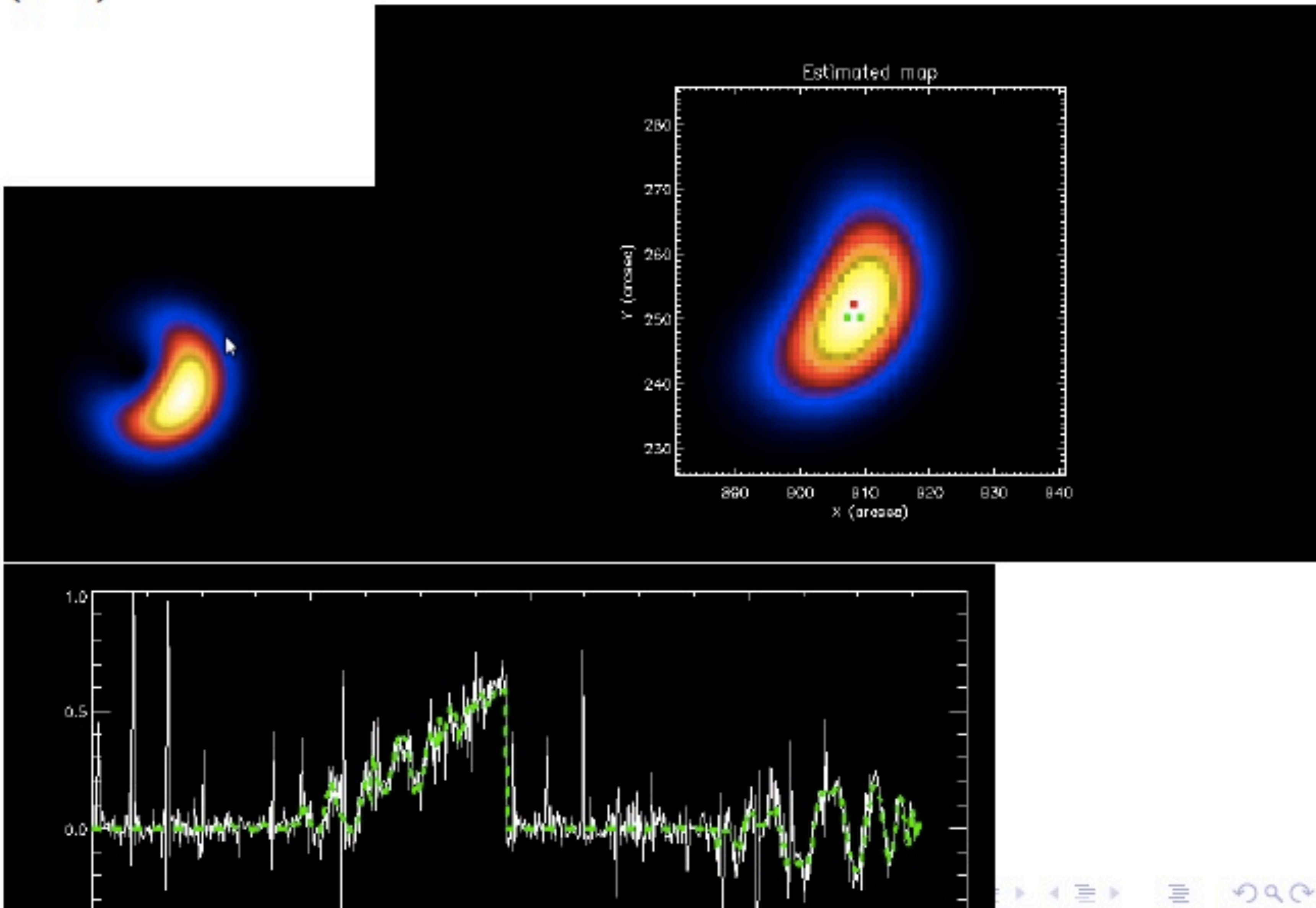
I set of N sources (ellipses or loops)

SMC

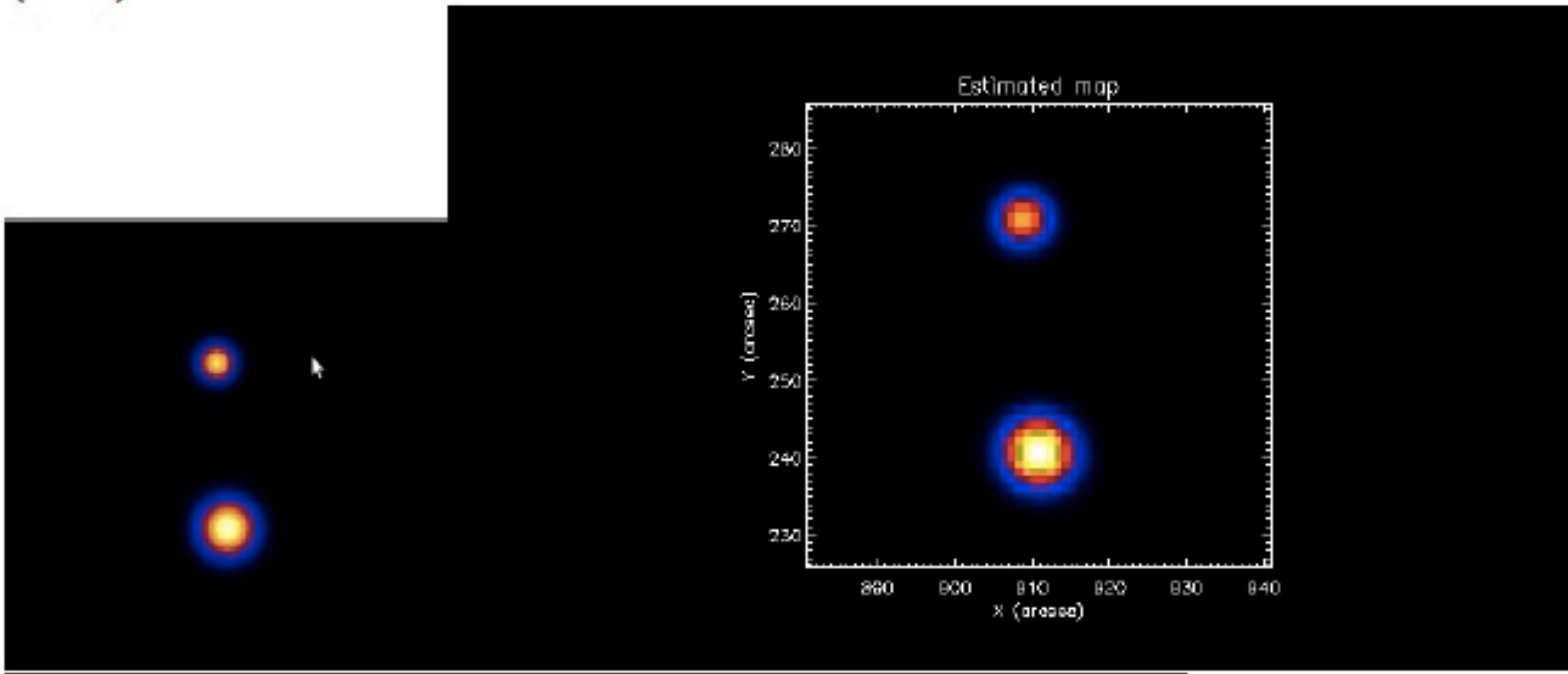
algorithm:

1. generate P random candidate solutions using the prior
2. compute the visibilities corresponding to each candidate
3. assign a weight to each candidate according to the likelihood
4. discard low-weight candidates and reproduce high weight ones
5. let the candidate evolve according to an MCMC process

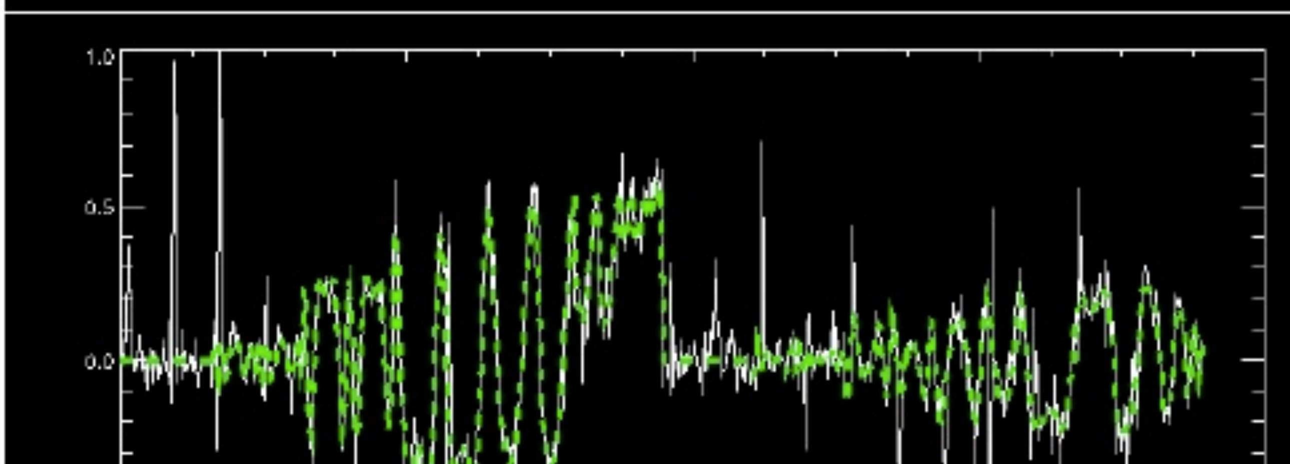
(true)


Estimated map
280
270
260
250
240
230
Y (arcsec)
890
900
910
920
930
940
X (arcsec)
1.0
0.5
0.0

(true)



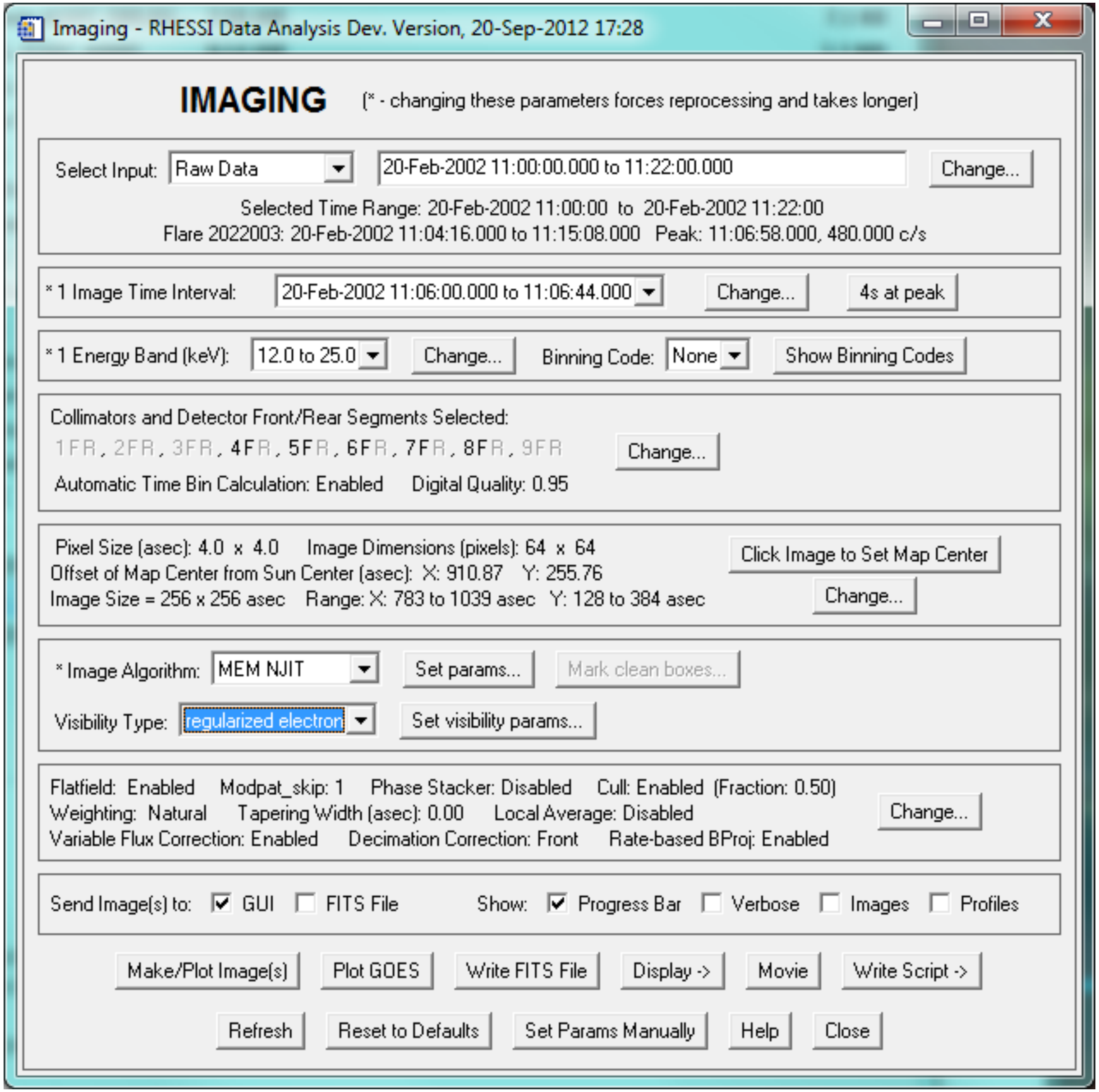
Estimated map
280
270
260
250
240
230
Y (arcsec)
890
900
910
920
930
940
X (arcsec)



1.0
0.5
0.0

imaging spectroscopy

electron maps



new release in progress: non-uniform photon energy channels allowed

source dimension

$$\frac{L(E)}{2} = \frac{L_0}{2} + \frac{1}{Kn}\sqrt{\frac{2}{(\delta-3)(\delta-5)}}\,E^2.$$

volume: $V_0 = \frac{\pi W^2 L_0}{4}$

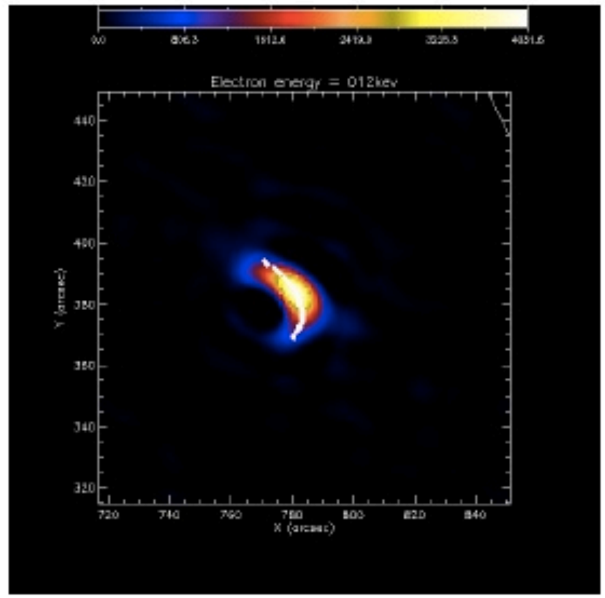
number of particles: $\mathcal{N} = n\,V_0$

acceleration rate: $\eta(E_0) = \frac{1}{\mathcal{N}}\frac{d\mathcal{N}}{dt}(\geq E_0)$

filling factor: $f = \frac{V_{\rm emit}}{V} = \frac{\rm EM}{n^2 V}$

Event No.	L_0 (arcsec)	W (arcsec)	V_0 (100 arcsec^3)	n (10^{11} cm^{-3})	$\mathcal{N}$ (10^{37})	η (20 keV) (10^{-3} s^{-1})	f
1	18.6	7.0	7.2	1.5	4.1	6.5	0.45
2	16.3	6.9	6.2	1.4	3.2	14.5	0.83
3	16.7	7.3	7.0	4.4	11.7	4.0	0.04
4	16.6	7.3	7.0	4.8	12.8	7.3	0.11
5	16.6	8.2	8.7	10.5	34.9	3.3	0.03
6	11.9	5.9	3.3	4.9	6.0	0.6	0.02
7	10.4	6.0	3.0	1.8	2.0	12.1	0.44
8	17.8	6.9	6.4	2.6	7.1	24.1	0.90
9	18.8	6.6	6.5	2.9	7.7	23.1	1.05
10	15.1	6.0	4.2	2.9	5.4	13.8	0.72
11	16.0	5.7	4.1	1.9	3.1	27.8	1.95
12	10.3	6.6	3.5	5.1	6.7	4.9	0.08
13	9.9	6.5	3.3	4.6	5.7	4.1	0.18
14	21.5	5.3	4.8	1.5	2.8	1.4	0.13
15	17.4	6.3	5.4	0.8	1.7	1.7	1.03
16	17.8	6.4	5.8	2.3	5.1	0.3	0.18
17	11.0	6.2	3.3	3.9	5.0	2.9	0.05
18	9.9	6.3	3.1	3.2	3.8	7.0	0.22
19	19.9	6.2	6.1	11.1	25.7	13.6	0.02
20	14.5	6.1	4.2	5.2	8.3	23.4	0.10
21	9.9	6.1	2.9	2.2	2.4	16.5	0.53
22	12.4	6.0	3.6	1.7	2.3	5.2	0.26

continuity equation



$$F(s,E) = F_+(s,E) + F_-(s,E)$$

$$\pm \frac{\partial}{\partial s} F_\pm(s,E) - \frac{\partial}{\partial E}\left(-\frac{dE}{ds} F_\pm(s,E)\right) = S(s,E)$$

the physics is in:

- the energy loss rate $\frac{dE}{ds}(n,T)$
- the injection term $S(s,E)$

continuity equation

some definitions:

$$\Phi_{\pm}(s,E) := -\frac{dE}{ds}F_{\pm}(s,E) \qquad A(s,E) := -\frac{dE}{ds}\frac{S(s,E)}{\sqrt{2mE}}$$

$$x(E) := x_0 - \int_{E_0}^{E}\frac{1}{\frac{dE}{ds}}dE' \qquad \Phi(s,E) = \Phi_{+}(s,E) + \Phi_{-}(s,E)$$

wave equation for the electron maps:

$$-\frac{\partial^2}{\partial x^2}\Phi(s,E(x)) + \frac{\partial^2}{\partial s^2}\Phi(s,E(x)) = 2\frac{\partial}{\partial x}A(s,(E(x))$$

initial conditions:

$$\lim_{x\to\infty}\Phi(x,s\pm x) = 0 \qquad \lim_{x\to\infty}\partial_x\Phi(x,s\pm x) = 0 \qquad \forall s$$

continuity equation

$$\Box G(s,s';x,x') = -\partial_x \delta(s,s';x,x')$$

two solutions:

$$G(s,s';x,x') = \delta_{\pm}(x - x' \mp |s - s'|)$$

but just one (δ_-) is coherent with the initial conditions. therefore:

$$\Phi(s, E(x)) = (\delta_- * 2A)(x, s)$$

$$F(s, E(x)) = \sqrt{\frac{2}{m}} \frac{1}{\frac{dE}{ds}} \int \delta(x - x' + |s - s'|) \frac{dE}{ds} \frac{S(s', E(x'))}{\sqrt{E(x')}} dx' ds'$$

lhs: measurements

rhs: models (for the energy loss rate and for the injection term)

continuity equation

15–Apr–2002

Time[UT]	Mod	$E_0[keV]$	$n[cm^{-3}]$	$h_s[cm^{-5}keV^{-1}s^{-1}]$	χ^2
00:03:00-00:06:00	Mod 1	7.7 ± 0.2	$(8.0 \pm 0.8) \times 10^{10}$	$(1.6 \pm 0.3) \times 10^{30}$	0.80
	Mod 2	2.5 ± 0.1	$(1.81 \pm 0.16) \times 10^{11}$	$(4.7 \pm 0.8) \times 10^{30}$	1.04
	Mod 3	7.9 ± 1.9	$(7.86 \pm 0.12) \times 10^{9}$	$(4.2 \pm 0.5) \times 10^{30}$	0.82
	Mod 4	0	$(3.4 \pm 0.3) \times 10^{10}$	$(9.52 \pm 1.19) \times 10^{30}$	3.58
00:06:00-00:09:00	Mod 1	8.2 ± 0.3	$(6.3 \pm 0.6) \times 10^{10}$	$(1.04 \pm 0.21) \times 10^{30}$	0.33
	Mod 2	3.2 ± 0.1	$(1.42 \pm 0.11) \times 10^{11}$	$(3.3 \pm 0.5) \times 10^{30}$	0.55
	Mod 3	3.5 ± 0.8	$(6.99 \pm 0.12) \times 10^{9}$	$(3.5 \pm 0.4) \times 10^{30}$	0.43
	Mod 4	0	$(2.30 \pm 0.19) \times 10^{10}$	$(7.2 \pm 0.8) \times 10^{30}$	2.45
00:09:00-00:12:00	Mod 1	4.9 ± 1.2	$(1.6 \pm 0.3) \times 10^{10}$	$(5.1 \pm 2.3) \times 10^{28}$	0.65
	Mod 2	3.2 ± 0.6	$(4.5 \pm 0.7) \times 10^{10}$	$(3.45 \pm 1.11) \times 10^{29}$	0.70
	Mod 3	0.2 ± 0.1	$(4.47 \pm 0.11) \times 10^{9}$	$(1.00 \pm 0.24) \times 10^{30}$	0.79
	Mod 4	0	$(6.2 \pm 0.5) \times 10^{9}$	$(1.4 \pm 0.3) \times 10^{30}$	1.21