Empirical Determination of the Energy Loss Rate of Accelerated Electrons in a Well-Observed Solar Flare

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Plan of the talk

- Theoretical models for electron motion during flares.
  - Energy loss rate
  - Source terms

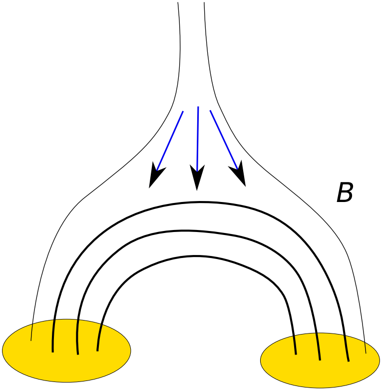
- Comparison with a well observed flare
  - Analysis of energy loss rate in energy domain
  - Spatial and spectral resolved analysis of the source term

Bibliography

- G. Torre, et all. APJ **751 (2)**, 129 (2012)
- G. Torre, et all. in preparation

Electron motions and continuity equation

- Discarding backreaction on $\vec{B}$
- Discarding quantum effects
- Assuming constant $\vec{B}$
- Electrons move freely along field lines
- Rotate in the orthogonal plane
- Consider the motion along the field



Continuity equation

For the electron Flux in stationary regimes we have

$$\pm\frac{\partial F(E,s)}{\partial s}+\frac{\partial}{\partial E}\left(F(E,s)\left|\frac{dE}{ds}\right|\right)=S(E,s)$$

- **Energy Loss rate**

  $$\left|\frac{dE}{ds}\right|$$

  due to Coulomb collisions

- **Source term**

  $$S(E,s)$$

  injection term due to magnetic reconnection occurring above the loop

Energy loss term

It models energy loss due collisions with background particles.

In order to have an expression for such a term we need to use:

- **Pure Coulomb collisions** (discarding Bremsstrahlung)
- **Rutherford cross section** with a fixed screening angle.
- $e - e$ scattering is 2000 times more efficient.
- Considering some **kinetic background distributions** (equilibrium)

- **Maxwell distribution**, at fixed T *[Spitzer]*

$$f_T(\mathbf{v}) = e^{-\frac{\mathbf{v}^2}{w_T^2}}$$

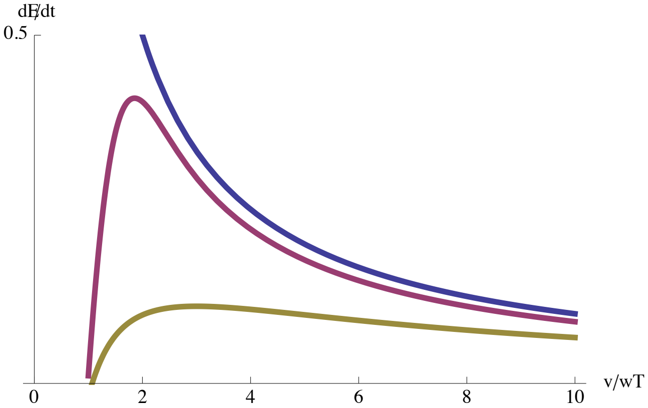
- **Cold target**

$$f_T \to f_0$$

- Maxwell dis. with a **coherent motion** $\vec{w}_0 \parallel \vec{v}$ **(return currents)**

$$f_{T,\mathbf{w}_0}(\mathbf{v}) = e^{-\frac{(\mathbf{v}-\mathbf{w}_0)^2}{w_T^2}}$$

Energy loss rate: model comparison



$\frac{dE}{dt}$ is linear wrt. background distributions. $\Longrightarrow$ different effects can be combined.

Just to impress you.....

$$\frac{dE}{dt} = -\frac{Kn}{w_T}\left[\left(\frac{w_T}{v+w_0} - 2\frac{w_0\, w_T}{v+{w_0}^2}\right)\mathrm{Erf}\left(\frac{v+w_0}{w_T}\right)\right.$$
$$\left. - e^{-\frac{v+{w_0}^2}{w_T^2}}\frac{4}{\sqrt{\pi}}\left(1-\frac{w_0}{v+w_0}\right)\right]$$

- v particle velocity
- w_T thermal velocity
- w_0 velocity of coherent motion (velocity of return currents).

[Codispoti et al. in preparation]

Source term

Extended source (injection) terms

- If the energy loss term is positive
- Assuming $F(E,s) \to 0$ for $E \to \infty$

An **unique solution** for the continuity equation exists

$$F(E,s) = \frac{2}{\left|\frac{dE}{ds}\right|} \int_{-\infty}^{\infty} \left|\frac{dE}{ds}\right| \; S\left(E(x + |s - s_0|), s_0\right) \; ds_0$$

where

$$x(E) = x_0 + \int_{E_0}^{E} \frac{1}{\left|\frac{dE}{ds}\right|} dE'$$

Extended source terms, a reasonable expression for an extended source is

$$S(E,s) = C \left(\frac{E_0}{E}\right)^{\delta} e^{-\frac{(s-s_0)^2}{2\sigma}}$$

[Reznikova et al. 2009]

The Continuity Equation on Flare Images

$$\frac{\partial F(E,s)}{\partial s} + \frac{\partial}{\partial E}\left(F(E,s)\frac{dE}{ds}\right) = S(E,s)$$

Assumptions:

$$F(x,y,z;E) = \overline{F}(x,y;E) \tag{1}$$

$$\left|\frac{dN(s)}{ds}F(E,s)\right| << \left|N(s)\frac{\partial F(E,s)}{\partial s}\right| \tag{2}$$

$$g(E,s) = N(s)\overline{F}(E,s) \qquad (electrons \quad cm^{-4}s^{-1}keV^{-1})$$

$$h(E,s) = N(s)S(E,s) \qquad (electrons \quad cm^{-5}s^{-1}keV^{-1})$$

[Piana et al. 2008]

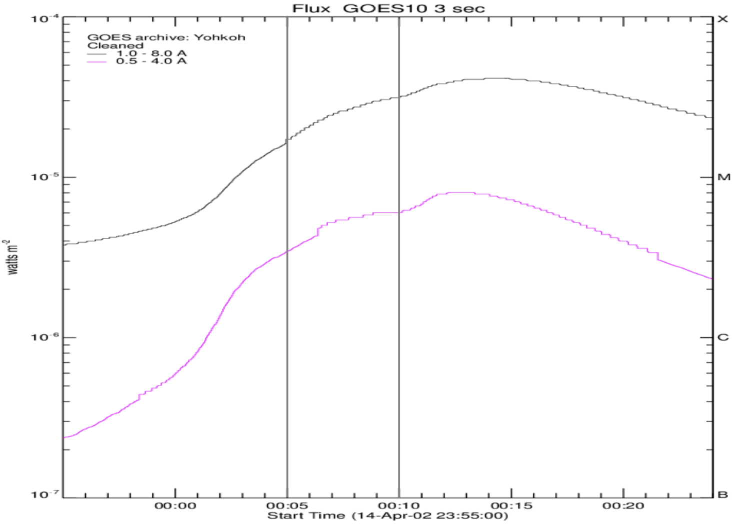
The Continuity Equation on Flare Images (2)

$$\frac{\partial g(E,s)}{\partial s} + \frac{\partial}{\partial E}\left(g(E,s)\frac{dE}{ds}\right) = h(E,s)$$

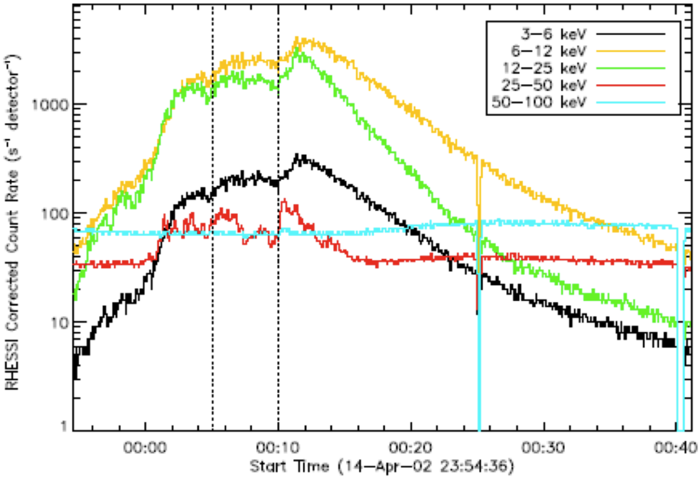
Solving the continuity Equation in terms of $R(s,E)$:

$$R(E,s) \equiv -\frac{1}{g(E,s)}\int_E^{\infty}\frac{\partial g(E,s)}{\partial s}dE = -\frac{dE}{ds}(E,s) - \frac{1}{g(E,s)}\int_E^{\infty}h(E,s)dE$$

2002 April 15



2002 April 15 (2)



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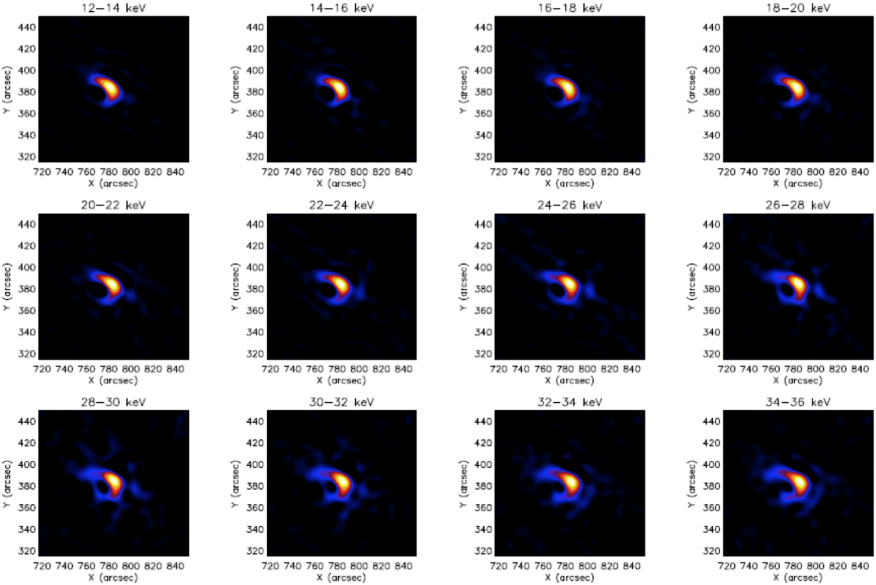
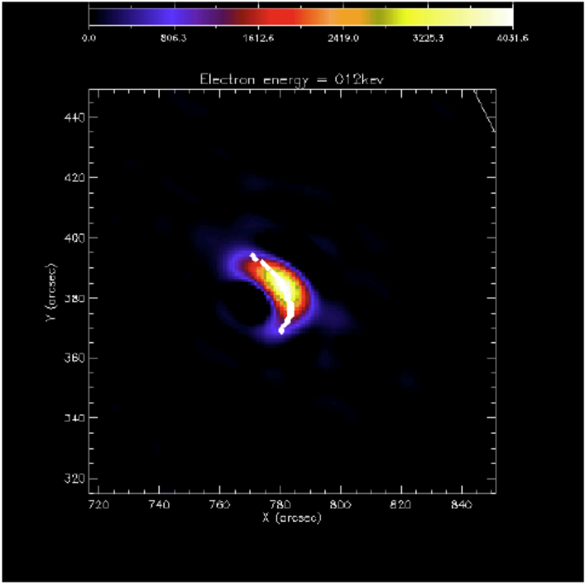
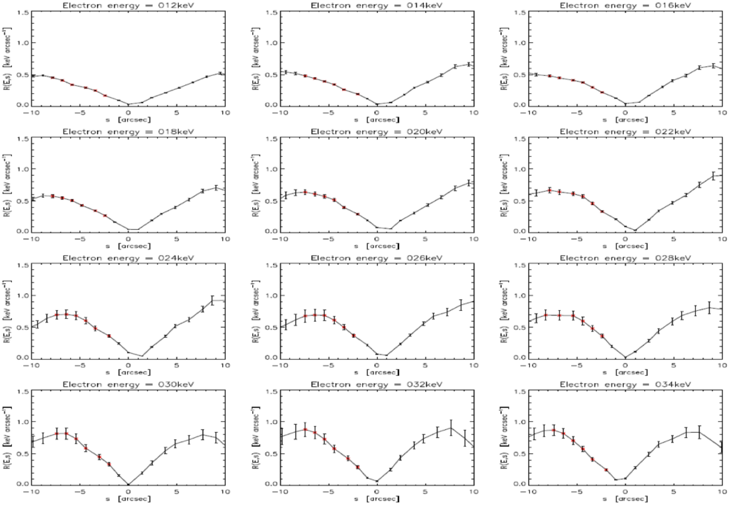


2002 April 15 (4)

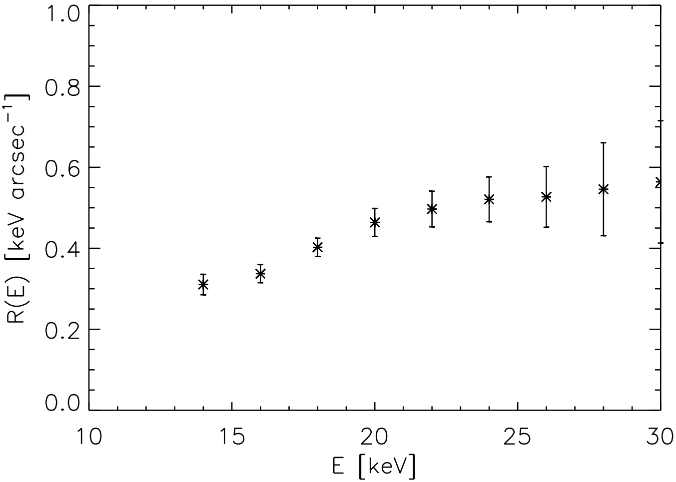


Application to Observations

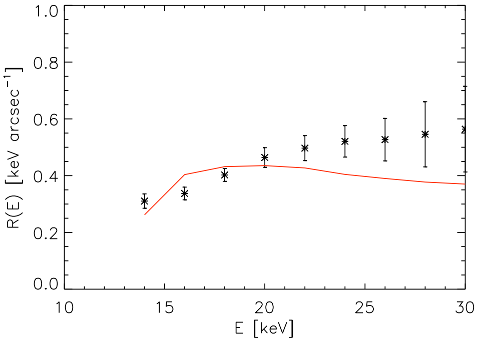
$$R(E,s) \equiv -\frac{dE}{ds}(E,s) - \frac{1}{g(E,s)}\int_E^{\infty} h(E,s)\,dE = -\frac{1}{g(E,s)}\int_E^{\infty} \frac{\partial g(E,s)}{\partial s}\,dE$$



Application to Observations (2)



Spitzer Model

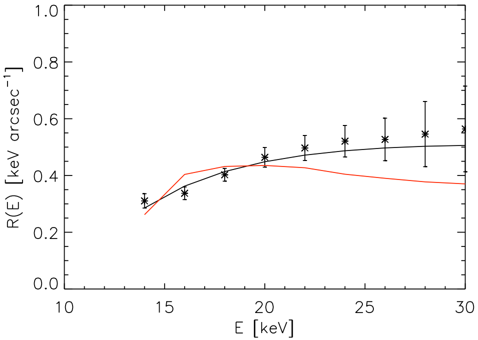


Fit Results

$n = (1.11 \pm 0.06) \times 10^{10} cm^{-3}$

$\eta \approx 6.3 \times 10^{-3} s^{-1}$

Return Currents Model



Fit Results

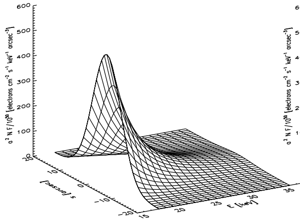
$n = (6.98 \pm 0.06) \times 10^{10} cm^{-3}$
$E_0 = (9.9 \pm 3.2) keV$
$\eta \approx 1.34 \times 10^{-5} s^{-1}$

Electron Flux

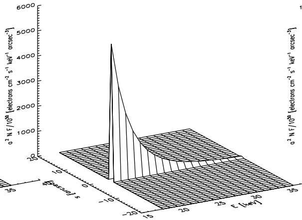
With this model the **injected electron flux** has the same magnitude as the **return electron flux**.

Spatial and spectral resolved analysis of the source term

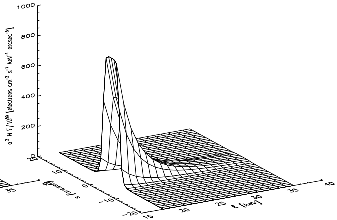
$$F(E,s) = \frac{2}{\left|\frac{dE}{ds}\right|} \int_{-\infty}^{\infty} \left|\frac{dE}{ds}\right| \; S\left(E(x+|s-s_0|), s_0\right) \; ds_0$$



$$\frac{S_0}{E^{\delta_0}} e^{-\frac{s}{2\sigma^2}}$$

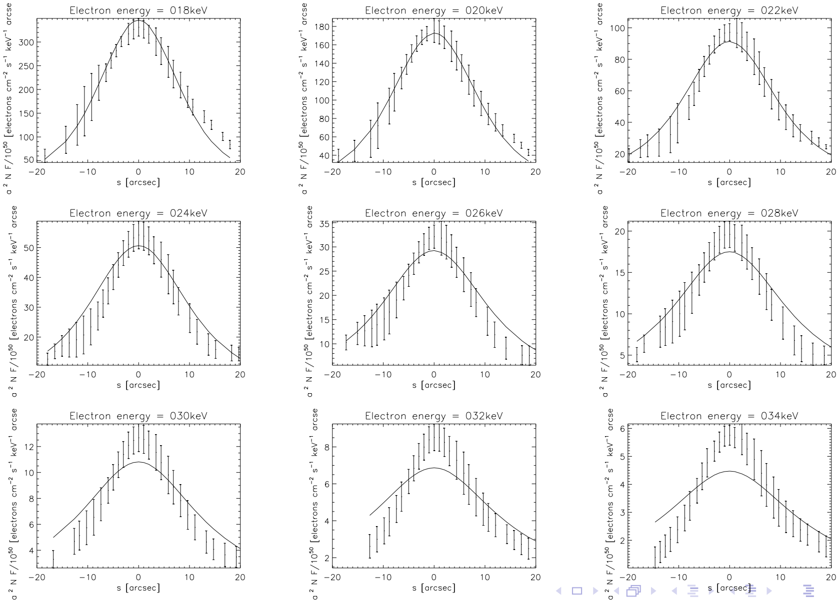


$$\frac{S_0}{E^{\delta_0}} \delta(s)$$

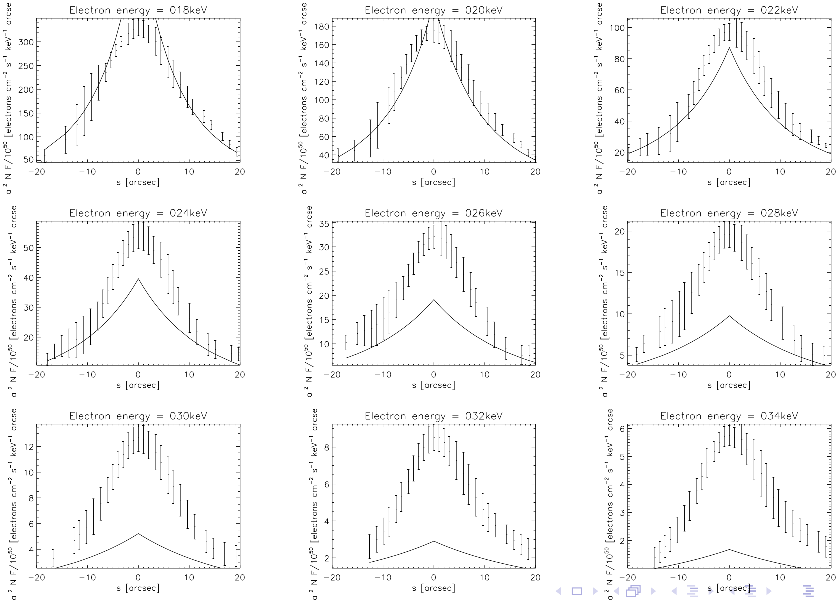


$$\frac{S_0}{E^{\delta_0}} \theta(\sigma - |s|)$$

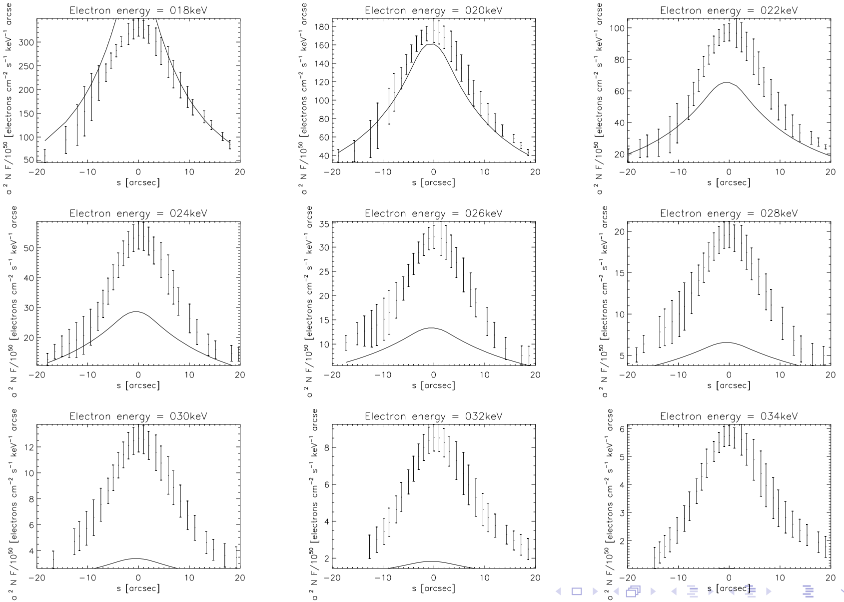
Gaussian Source



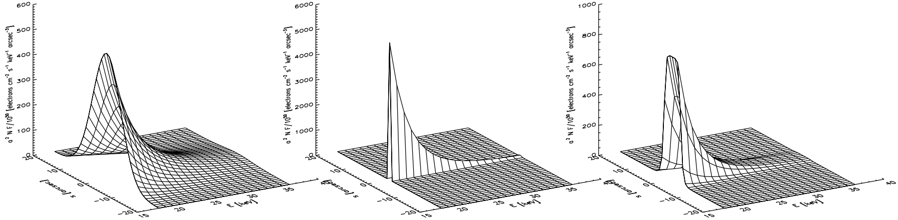
Pointwise Source



Square Source



Results



	$S_0 \times 10^{12}$	σ $[arcsec]$	$n \times 10^{10} [cm^{-3}]$	δ_0
Gauss	(4.1 ± 0.5)	(5.4 ± 0.3)	(3.99 ± 0.15)	(5.51 ± 0.05)
Point	(4.1 ± 0.6)	-	(2.11 ± 0.04)	(7.07 ± 0.05)
Square	(4.1 ± 0.8)	(3.19 ± 0.05)	(1.73 ± 0.09)	(7.73 ± 0.07)

Summary

- Continuity equations
- Models for the energy loss rates and for the source terms
- Comparison with observed data to extract the parameters
- Models agree with observation

Open Questions

- Does it work also for less nicer flares?

Statistic Study of Loop-Structured Flares with Coronal X-ray Sources

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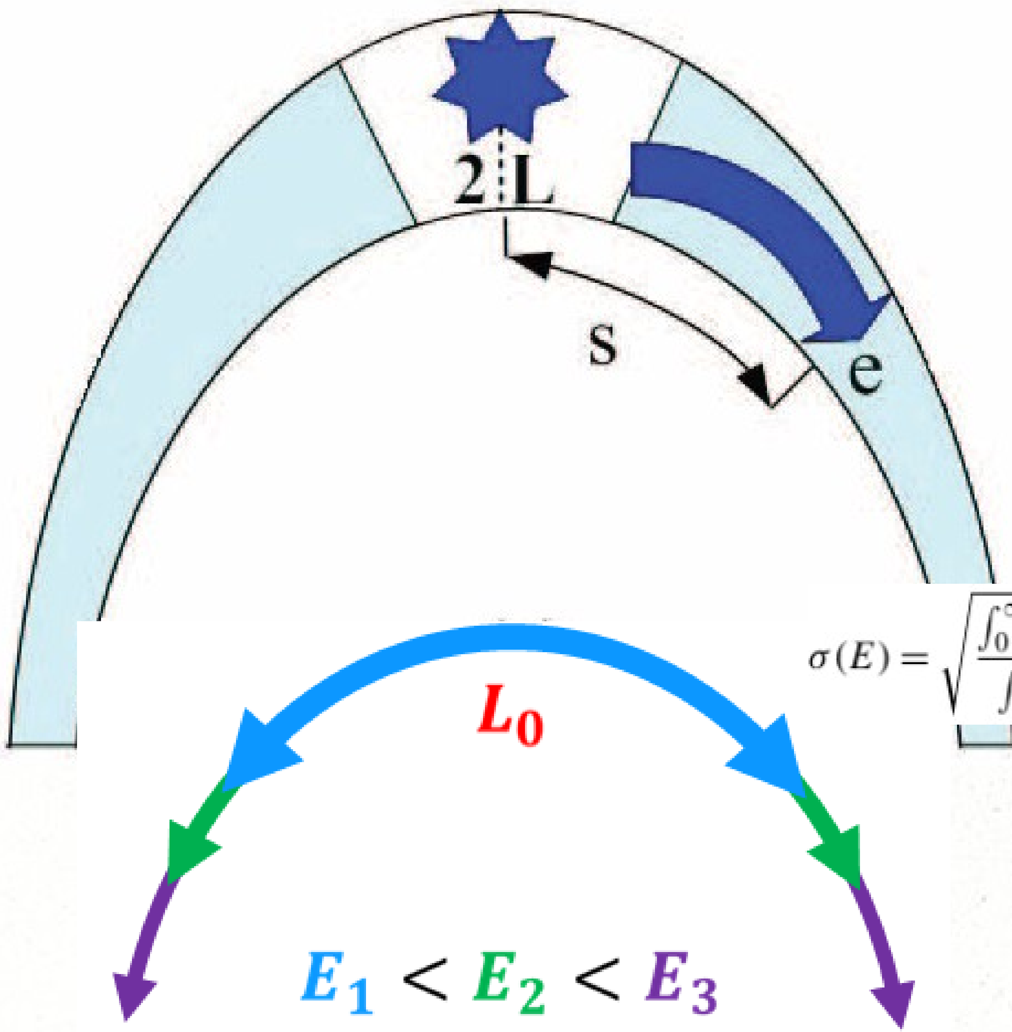
Graz, Oct 2012

Key Words:
Acceleration in flare loops
size of acceleration region
coronal loop density
specific acceleration rate
filling factor

Collisional Model with Extended **Tenuous** Acceleration

Xu et al. 2008
Prato et al. 2009
Kontar et al. 2011
Guo et al. 2012



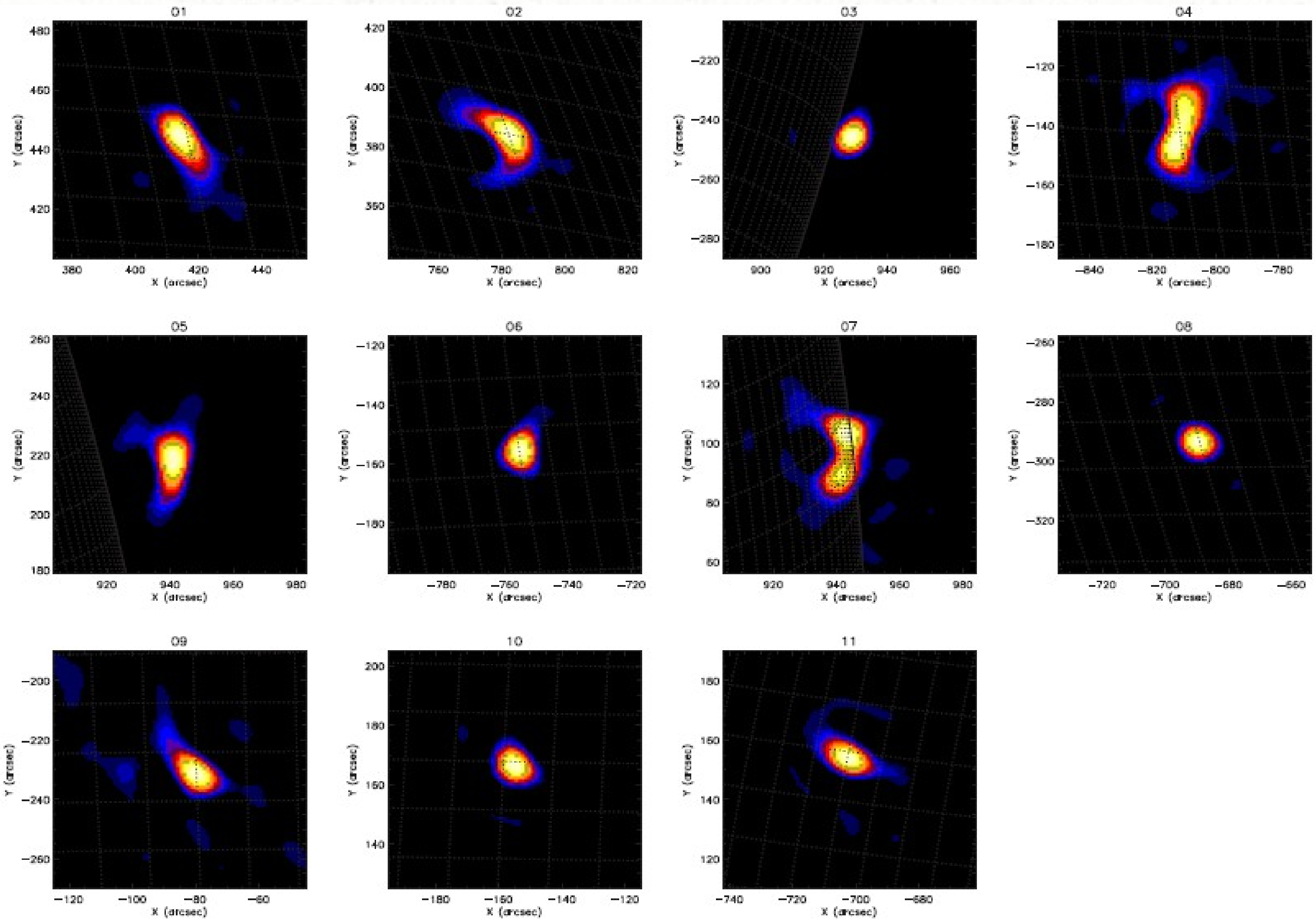
$$L(\epsilon) = L_0 + A_1 \epsilon^2$$

$$F(E,s) \sim \frac{E}{(E^2 + 2Kns)^{(\delta+1)/2}} \tag{1}$$

$$\sigma(E) = \sqrt{\frac{\int_0^\infty s^2 F(E,s)\,ds}{\int_0^\infty F(E,s)\,ds}} = \sqrt{\frac{2}{(\delta-3)(\delta-5)}}\,\frac{E^2}{Kn} \tag{2}$$

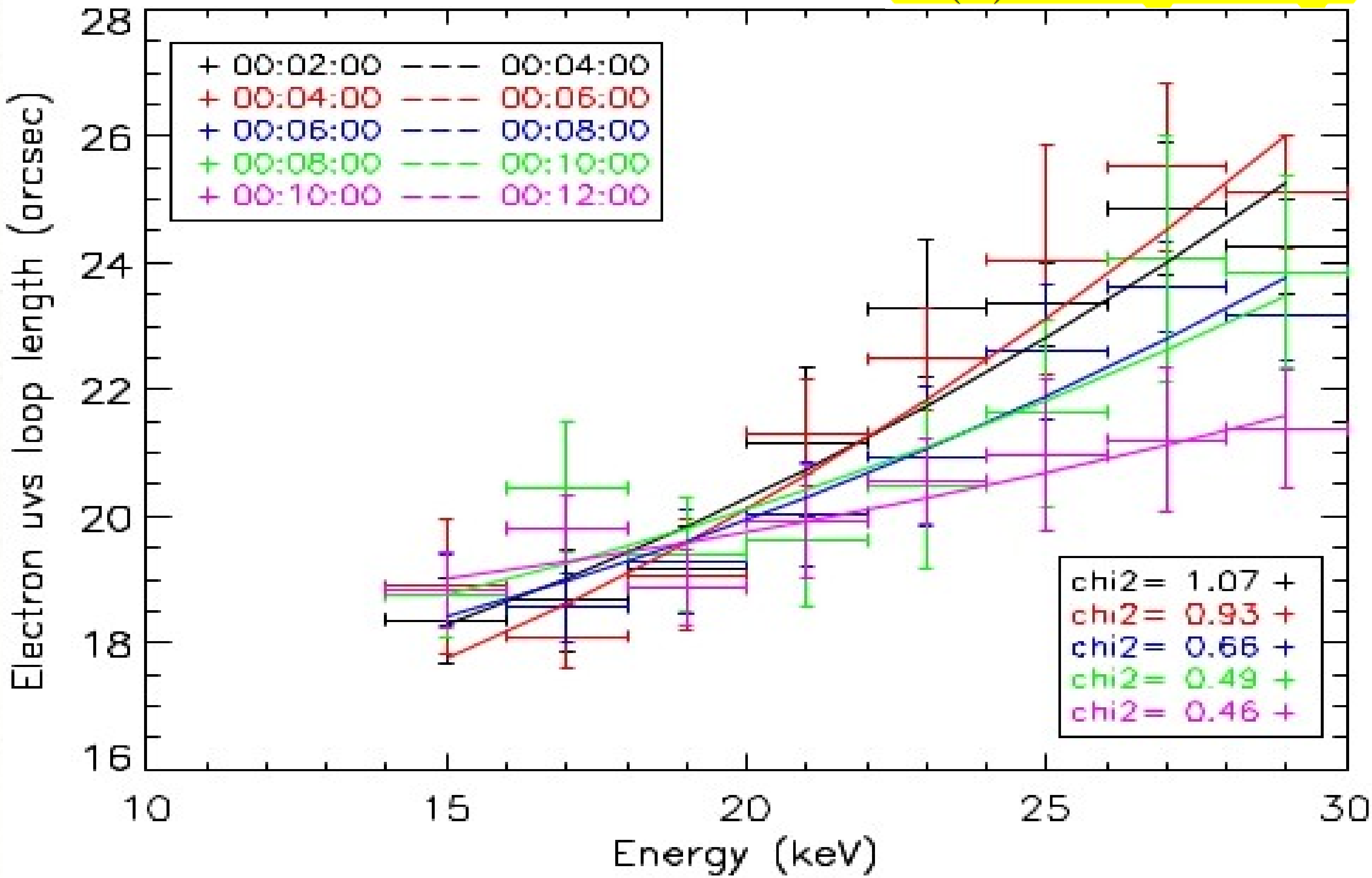
$$\frac{L(E)}{2} = \frac{L_0}{2} + \frac{1}{Kn}\sqrt{\frac{2}{(\delta-3)(\delta-5)}}\,E^2 \tag{3}$$

Loop-structured Flares



Quadratic Function

$$L(\epsilon) = L_0 + A_1\epsilon^2$$



The data fits well!

But…the model is not so consistent

The acceleration region, having the same density as the rest of the loop, is however decoupled from Coulomb collisions

Event No.	L_0 (arcsec)	W (arcsec)	V_0 (100 arcsec^3)	n (10^{11} cm^{-3})	$\mathcal{N}$ (10^{37})	η (20 keV) (10^{-3} s^{-1})	f
1	18.6	7.0	7.2	1.5	4.1	6.5	0.45
2	16.3	6.9	6.2	1.4	3.2	14.5	0.83
3	16.7	7.3	7.0	4.4	11.7	4.0	0.04
4	16.6	7.3	7.0	4.8	12.8	7.3	0.11
5	16.6	8.2	8.7	10.5	34.9	3.3	0.03
6	11.9	5.9	3.3	4.9	6.0	0.6	0.02
7	10.4	6.0	3.0	1.8	2.0	12.1	0.44
8	17.8	6.9	6.4	2.6	7.1	24.1	0.90
9	[illegible]	[illegible]	6.5	2.9	7.7	23.1	1.05
10	15.1	6.0	4.2	2.9	5.4	13.8	0.72
11	16.0	5.7	4.1	1.9	3.1	27.8	1.95
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.08
13	9.9	6.5	3.3	[illegible]	5.7	4.1	0.18
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.13
15	17.4	6.3	5.4	0.8	1.7	1.7	1.03
16	[illegible]	6.4	[illegible]	[illegible]	[illegible]	0.3	0.18
17	11.0	6.2	[illegible]	3.9	[illegible]	2.9	0.05
18	9.9	6.3	3.1	3.2	3.8	7.0	0.22
19	19.9	6.2	6.1	11.1	25.7	13.6	0.02
20	14.5	6.1	4.2	5.2	8.3	23.4	0.10
21	9.9	6.1	2.9	2.2	2.4	16.5	0.53
22	12.4	6.0	3.6	1.7	2.3	5.2	0.26
Geometric Mean	14.5	6.4	4.7	2.9	5.4	6.0	0.2
×/÷	1.3	1.1	1.4	1.9	2.2	3.4	3.9

Thanks a lot for your attention!