Expectation maximization method in X-ray solar astronomy

Federico Benvenuto

University of Genova

benvenuto@dima.unige.it

Graz, October 2-3, 2012

Statistical approach to imaging problem formulation

Starting from count modulation profiles, the reconstruction problem is to find the image f such that

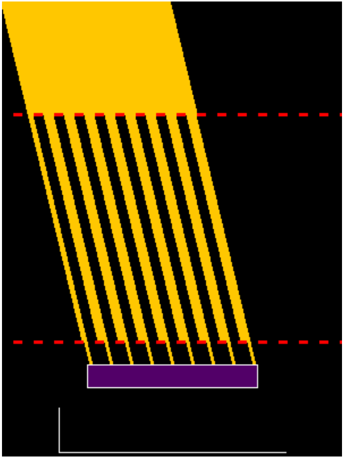
The model

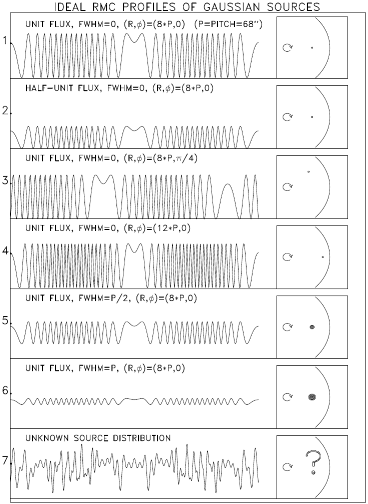
$$Hf = g$$

where

- g is the detected count modulation profiles.
- H is a linear operator which describes the transformation between images and profiles provided by the RMCs.

Matrix H mimicks Rhessi production of count modulation profiles





Statistical approach

The vector g can be consider a realization of a Poisson random variable Y with expected value Hf.

The probability density function of variable Y

Poisson probability

$$p(g; f) = \prod_{i \in R} \frac{e^{-(Hf)_i}}{g_i!} (Hf)_i^{g_i}$$

yields the Likelihood $L(f; g)$ of the problem.

Maximum likelihood method obtains the solution of f when the probability function reaches its maximum value.

Constrained minimization

We are interested in non negative solutions.

We minimize the C-statistics ($\simeq$ negative logarithm of the Likelihood)

C-statistics

$$C(g,f) = \sum_{i=0}^{N} g_i \log \frac{g_i}{(Hf)_i} + (Hf)_i - g_i$$

under non negative constraint, i.e.

Constrained minimization problem

$$\min C(g,f) \quad | \quad f \geq 0$$

Expectation maximization algorithm

- The necessary and sufficient condition for f to be the solution of the constrained C-statistics minimization is Karush-Khun-Tucker (KKT) conditions. They lead to a fixed point equation iteratively solved by the EM algorithm

EM algorithm

$$f^{k+1} = \frac{f^k}{H^T \mathbf{1}} H^T \left(\frac{g}{Hf^k} \right)$$

- Property 1 : This algorithm converges to a non negative solution of the problem.
- Property 2 : At each iteration the total number of expected counts is equal to the total detected counts.
- Some regularization is required.

Regularization specifically coinceived for Rhessi imaging

- Classical stopping rules do not efficiently work for Rhessi imaging:

$$r(f, g) \leq \mathbb{E}_Y \ r(f, g)$$

  where $r(f, g)$ is some approximation of the C-statistics.
- This approach does not efficiently work since it does not take into account the positive constraint on the solution !
- We developed a new rule to regularize EM for Rhessi imaging by taking:

$$r(f, g) = \left\| f \cdot H^T \left(\frac{g}{Hf} - 1 \right) \right\|^2$$

EM algorithm: applications

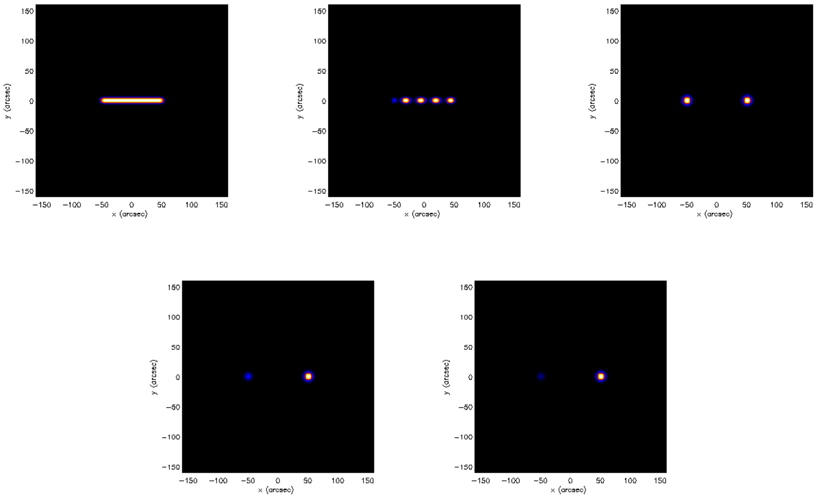


Figure: Annapolis synthetic maps

EM algorithm: applications

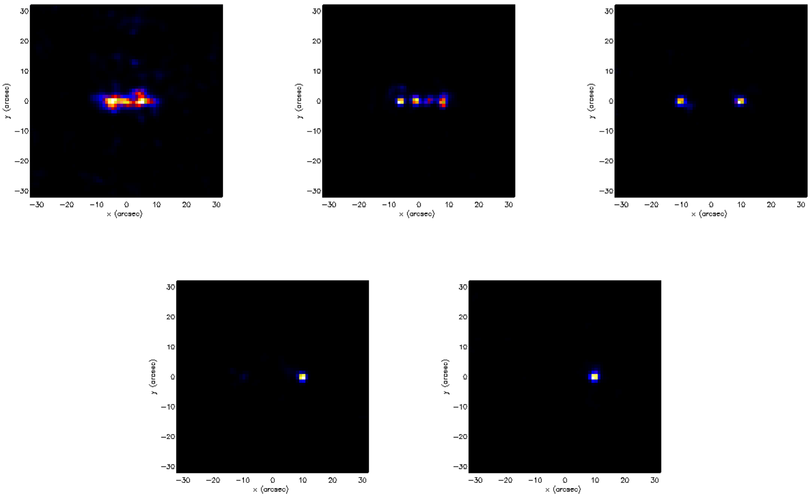


Figure: Low statistics: 10^3 counts per detector

EM algorithm: applications

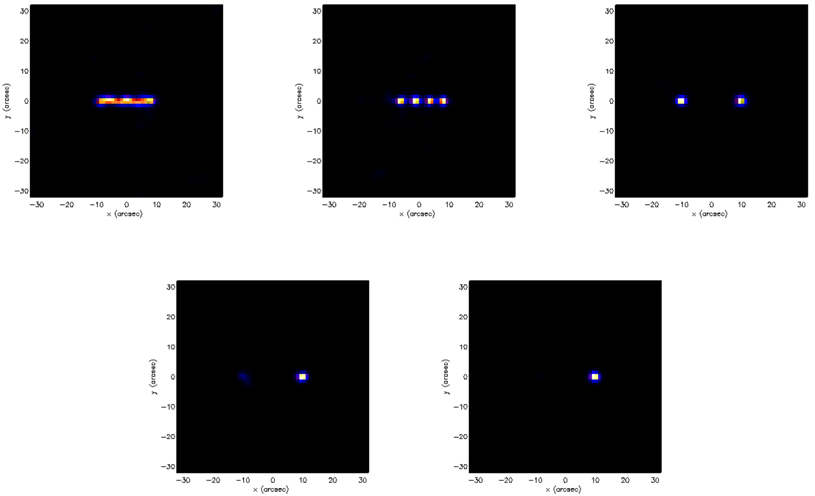


Figure: Medium statistics: 10^4 counts per detector

EM algorithm: applications

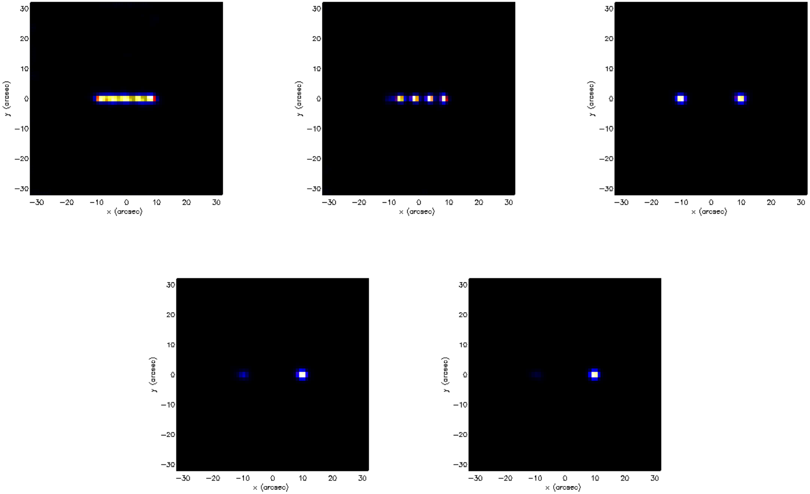


Figure: High statistics: 10^5 counts per detector

EM algorithm: applications

Numerical comparison with Pixon method:

LINE SOURCE WITH CONSTANT INTENSITY

CRITERIUM A: ORIENTATION (0 deg)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	-0.10	-0.61	3.84
EM	-0.06	-0.41	3.33

CRITERIUM B: Number of sources at FWHM (1)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	1	2	2
EM	1	3	2

CRITERIUM C:
Number of sources from local minima analysis and K=0.2 (1)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	1	3	2
EM	1	3	2

LINE SOURCE WITH VARYING INTENSITY

CRITERIUM A: ORIENTATION (0 deg)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	-0.10	0.01	0.11
EM	-0.04	-0.06	-0.96

CRITERIUM B: Number of sources at FWHM (4)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	4	4	3
EM	4	4	3

CRITERIUM C:
Number of sources from local minima analysis and K=0.2 (5)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	5	4	4
EM	4	4	4

EM algorithm: applications

Numerical comparison with Pixon method:

LINE SOURCE WITH CONSTANT INTENSITY

CRITERIUM D: FWHM (20.2 arcsec)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	19.81	18.31	14.47
EM	18,86	17.84	13.51

CRITERIUM E: FWHM in the orthogonal direction (1.35 arcsec)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	3.04	2.17	4.27
EM	2,08	1.94	3.45

CRITERIUM F: rms

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	0.6082	0.5297	0.7519
EM	0,4865	0.4895	0.6991

LINE SOURCE WITH VARYING INTENSITY

CRITERIUM D: FWHM (16.90 arcsec)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	17.82	16.58	17.13
EM	16.41	16.35	15.92

CRITERIUM E: FWHM in the orthogonal direction (1.35 arcsec)

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	3.79	2.81	3.10
EM	2.05	2.17	1.86

CRITERIUM F: rms

	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9
PIXON	0.7366	0.6521	0.7574
EM	0.5517	0.5553	0.6655

EM algorithm: applications

Numerical comparison with Pixon method:

TWO SOURCES: FLUX RATIO 1 - 5 - 10

	FLUX RATIO =1			FLUX RATIO =5			FLUX RATIO =10		
CRITERIUM G1: Distance of the first source from the simulated one (0 arcsec)									
	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9
PIXON	0.71	0.71	0.71	1.58	1.58	03.54	1.58	7.52	9.93
EM	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	3.54
CRITERIUM G2: Distance of the second source from the simulated one (arcsec)									
	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9
PIXON	0.71	0.71	0.71	1.58	1.58	03.54	1.58	7.52	9.93
EM	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.71	3.54
CRITERIUM H: separation between the two sources (20 arcsec)									
	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9
PIXON	22.02	19.00	19.00	18.00	19.00	24.02	19.00	28.02	27.66
EM	19.03	19.00	20.00	20.00	19.03	20.03	19.00	19.00	23.00

EM algorithm: applications

Numerical comparison with Pixon method:

TWO SOURCES: FLUX RATIO 1 - 5 - 10

CRITERIUM I: Orientation of the separation line (0 deg)

	FLUX RATIO =1			FLUX RATIO =5			FLUX RATIO =10		
	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9
PIXON	-2.6	0	0	0	0	-2.39	0	-2.05	-12.53
EM	3.01	0	0	0	-3.01	-2.86	0	0	0

CRITERIUM J: Ratio of the two peak intensities (1-5-10)

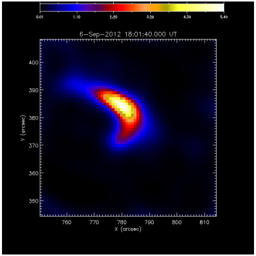
	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9
PIXON	0.95	0.93	1.56	5.77	9.34	121.12	7.72	74.44	471.86
EM	0.94	1.02	1.37	6.79	10.93	21.59	23.82	92.26	245.61

CRITERIUM K: Ratio of the two fluxes in the two circles centered in the two sources and with radius =d/2 (1-5-10)

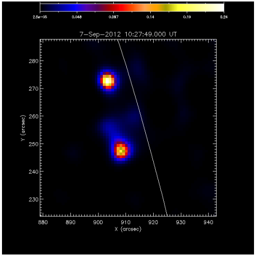
	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW	HIGH	MEDIUM	LOW
ALGORITHM	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9	DET 1-9
PIXON	1.01	1.02	1.00	5.14	5.57	11.53	11.65	16.96	42.69
EM	1.01	1.01	0.99	4.73	4.99	7.56	10.02	16.15	29.68

EM algorithm: applications with real data

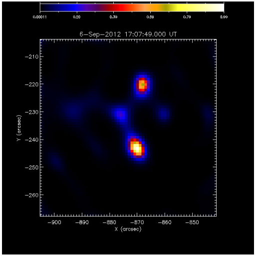
Reconstruction of three different events: a loop, two footpoints and two footpoints with a small loop-top.



(a) Apr 15 2002. Time: 00.06.00 00.08.00. Energy:12-14 keV.



(b) Feb 20 2002. Time: 11.05.58 11.06.41. Energy:25-30 keV.



(c) Jul 23 2002. Time: 00.30.00 00.32.00. Energy:100-300 keV.

EM algorithm in spectroscopy

Expectation maximization can be used for spectroscopy.

- g is a given count spectrum.
- f is the unknown electron spectrum.
- H is a matrix containing information about the Bremsstrahlung cross-section and the Detector Response Matrix (DRM).

Remark

DRM can take into account different aspects of Rhessi hardware. The inversion results can be different according to DRM choice.

Spectroscopy: blind tests

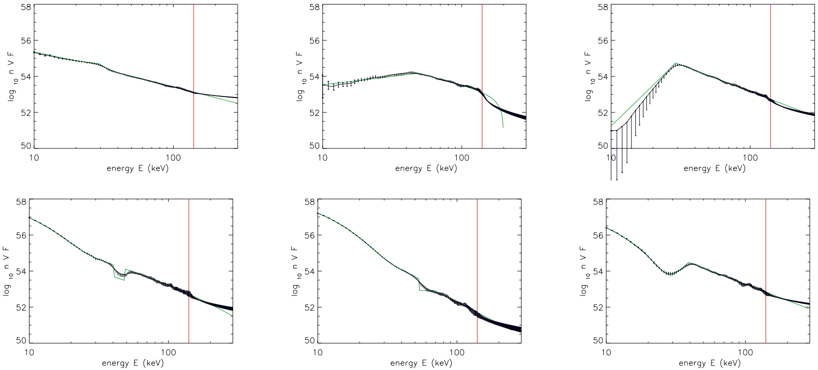


Figure: Six different synthetic spectra (green lines) and its reconstruction with EM. The red lines indicates the maximum energy of detected counts.

EM algorithm for electron flux maps

Our most recent advance is the application of EM for electron maps reconstruction.

- g is a given set of count modulation profiles detected from ϵ_{min} and ϵ_{max}.
- f is the unknown set of electron maps.
- H is a tensor containing information about energy and space.

Remark

In this context, the regularization of EM method provides smooth behaviour in energy and space. The resulting set of electron maps can be spatially integrated computing the total integrated flux spectrum.

EM algorithm for electron flux maps

April 15 2002 event: detector 3-8, 64 arcsec images. Energies of the reconstructed electron maps are from 14 up to 90 keV. Count modulation profiles are up to 52 KeV.

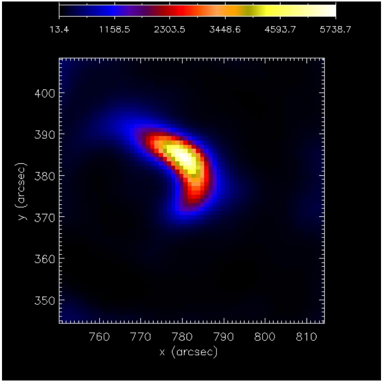


Figure: Electron maps

Rhessi imaging, spectroscopy and imaging spectroscopy

Conclusion

Expectation maximization is a powerful technique. Starting from count modulation profiles we can reconstruct:

- X-ray photon images.
- X-ray electron spectra.

Perspective

To reconstruct electron images by means of Expectation Maximization via counts

- X-ray electron images and spectra.

Thank you